Assignment - 2020

B.Sc (Hon) Physics Sem - II

GE-2 Linear Algebra

Max. Mar - 10

Do any fifteen Questions.

Q.1 (i) If x and y are vectors in R^n, then $|x \cdot y| \leq (\|x\|)(\|y\|)$.

(ii) If x and y are vectors in R^n, then $\|x+y\| \leq \|x\| + \|y\|$.

Q.2 Calculate projab and verify b- projab is othogonal to a.
where a= [2,1,5], b= [1,4,-3]

Q.3

$$-2x_1 + x_2 + 8x_3 = 0$$
$$7x_1 - 2x_2 - 22x_3 = 0$$
$$3x_1 - x_2 - 10x_3 = 0$$

use Gauss-Jordan method.

Q.4 Find the reduced row echelon from matrix B of the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & -3 \\ 0 & -2 & 5 \end{bmatrix}$$

Q.5 Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ -1 & -3 & 1 \end{bmatrix}$

Q.6 Determine weather the vector v=[2,2,-3] is in the row space of the matrix.

$$A = \begin{bmatrix} 4 & -1 & 2 \\ -2 & 3 & 5 \\ 6 & 1 & 9 \end{bmatrix}$$

Q. 7 Diagonalize the matrix $A = \begin{bmatrix} -2 & 4 & -6 \\ 3 & -3 & 6 \\ 3 & -4 & 7 \end{bmatrix}$.

Q. 8 Find all eigenvalues corresponding to ~~each~~ the matrix $A = \begin{bmatrix} 3 & 4 & 12 \\ 4 & -12 & 3 \\ 12 & 3 & -4 \end{bmatrix}$ and ~~eigenvalues~~ ~~eigenvectors~~ eigenvectors and eigenspace as a set of linear combinations of fundamental eigenvectors.

Q. 9 Prove that R^2 is a vector space using the operations $\oplus$ and $\odot$ given by

$$[x, y] \oplus [w, z] = [x + w + 1, y + z - 2]$$

and $$a \odot [x, y] = [ax + a - 1, ay - 2a + 2]$$

find the zero vector 0 in V and the additive inverse $-v$ for any vector v in V.

Q. 10 Show that the set of vectors of the form $[a, b, 0, c, a - 2b + c]$ in R^5 forms a subspace of R^5 under the usual operations.

Q. 11 Use Simplified span method to find a simplified general form for all the vectors in span (S), where $S = \{[1,1,1], [2,1,1], [1,1,2]\}$

Q. 12 Let $B = \{[2,3,0,-1], [-1,1,1,-1]\}$ and $S = \{[1,4,1,-2], [-1,1,1,-1], [3,2,-1,0], [2,3,0,-1]\}$.

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(i) Show that B is a maximal independent subset of S.

(ii) Calculate dim (span(S)).

(iii) Does span (S) = R^4? why or why not?

Q. 13 Show that the following is a linearly dependent subset of M_{22}:

$$\left\{ \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ -6 & 1 \end{bmatrix}, \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix}, \begin{bmatrix} 3 & -8 \\ 0 & 0 \end{bmatrix} \right\}.$$

Q. 14 Let $S = \{[1,2],[0,1]\}$ and $T = \{[1,1],[2,3]\}$ be two bases for R^2. Let $v = [1,5]$ and $w = [5,4]$.

(a) Find the coordinate vectors of v and w with respect to the basis T.

(b) What is the transition matrix $P_{S \leftarrow T}$ from the T-basis to S-basis.

(c) find the coordinate vector of v and w with respect to the S-basis, using $P_{S \leftarrow T}$.

Q. 15 Show that the mapping $T: R^3 \rightarrow R^3$ given by $T([x_1, x_2, x_3]) = [-x_1, x_2, x_3]$ is a linear operator.

Q. 16 Let $T: M_{22} \rightarrow R^2$ be the Linear transformation given by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [6a-b+3c-2d, -2a+3b-c+4d]$$

(a) find the matrix A_{BC} for T with respect to the standard bases B (for M_{22}) and C (for R^2).

(b) Use part (a) to find the matrix A_{DE} for T with respect to the ordered bases

$$D = \left(\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right) \text{ and } E = \{[-2, 5], [-1, 2]\}.$$

Q. 17 Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. define by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$ then find a basis for Ker(T) and a basis for range

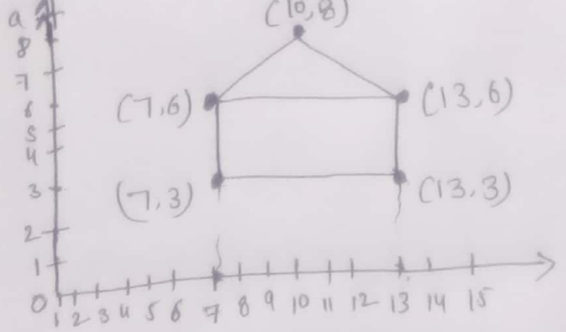
(a). range(T). Also, verify that dim(Ker(T)) + dim(range(T)) = dim V.

Q. 18 Let A be a fixed $n \times n$ matrix, and consider $T: M_{nn} \to M_{nn}$ given by $T(B) = AB - BA$.

(a) Show that T is not one-to-one. (b) Show that T is not onto.

Q. 19 Show that the mapping $T: M_{mn} \to M_{nm}$ defined by $T(A) = A^T$ is an isomorphism.

Q. 20



for the graphic in figure (above) use homogenous coordinates to find the new vertices after performing reflection about the line $y = 4x - 10$. then sketch the final figure that would result from this movement.