Regula Falsi Method

$$\boxed{x_{n+1} = \frac{x_n f(x_{n-1}) - x_{n-1} f(x_n)}{f(x_{n-1}) - f(x_n)}}$$

Algorithm

Step 1.) Take an interval $[x_0, x_1]$ such that $f(x_0) \cdot f(x_1) < 0$

Step 2.) Find $x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

Step 3) If $f(x_2) = 0$, then we take x_2 as the desired root and stop. Otherwise, we go to step 4.

Step 4.) If $f(x_0) \cdot f(x_2) < 0$, then we take x_2 as our new x_1 and go to step I.
Otherwise we take x_2 as our new x_0 and go to step 1 (i.e $f(x_0) \cdot f(x_2) > 0$)

Note:
In Regula falsi Method we have to check the sign $f(x_0) \cdot f(x_2)$ in each iteration
if $f(x_0) \cdot f(x_2) < 0$, then take $x_2 \leftrightarrow x_1$
if $f(x_0)\, f(x_2) > 0$, then take $x_2 \leftrightarrow x_0$

and each roots lies in the interval taken.

Eg: $x - e^{-x} = 0$

Sol^n:

$$f(x) = x - e^{-x}$$

As $f(0) < 0$ and $f(1) > 0$

$\Rightarrow$ Taking $x_0 = 0$ and $x_1 = 1$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0 \times 0.63212 - 1 \times (-1)}{0.63212 - (-1)}$$

$$\boxed{x_2 = 0.61270}$$

$$f(x_2) = 0.07081$$

$$f(x_0) \cdot f(x_2) < 0 \Rightarrow x_2 \leftrightarrow x_1$$

Now, $x_0 = 0$, $x_1 = 0.61270$

$$x_2 = \frac{0. \times 0.07081 - (0.61270) \times (-1)}{0.07081 - (-1)}$$

$$\boxed{x_2 = 0.57218}$$

$$f(x_2) = 0.00789 \quad (\because f(x_2) = 0.57218 - e^{-0.57218})$$

$$f(x_0) \cdot f(x_2) < 0 \Rightarrow x_2 \leftrightarrow x_1$$

$$x_0 = 0, \quad x_1 = 0.57218$$

Page No.
Date

Ques:- (i) $x^4 - x - 10 = 0$

(ii) $x^3 - x - 4 = 0$

find the interval in which the smallest +ve root of the following eq^n lies

(iii) $xe^x - 1 = 0$