Numerical Integration

To find, $I = \int_a^b f(x)\,dx$

let $a = x_0, x_1, x_2, \ldots, x_n = b$

where, $x_1 = x_0 + h$

$x_2 = x_0 + 2h$

$\vdots$

$x_n = x_0 + nh$

and $h = \dfrac{b-a}{n}$

Method I Trapezoidal Rule

The approximate area using the trapezoidal rule

$$\int_a^b f(x)\,dx \approx \frac{1}{2} h\left[(f_0 + f_n) + 2(f_1 + f_2 + \cdots + f_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$

and $f_i = f(x_i)$

Method 2 Simpson Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(f_0 + f_n) + 2(f_2 + f_4 + \cdots + f_{n-2}) + 4(f_1 + f_3 + f_5 + \cdots + f_{n-1})\right]$$

let $n =$ even

Que Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using

(i) Trapezoidal Rule

(ii) Simpson's Rule

Sol^n Let $n = 6$

$h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

$h = 1$

$f(x) = \frac{1}{1+x^2}$

$x_0 = 0$	$f(x_0) = \frac{1}{1+0^2} = 1$
$x_1 = x_0 + h = 1$	$f(x_1) = \frac{1}{1+1^2} = 1/2 = 0.5$
$x_2 = x_0 + 2h = 2$	$f(x_2) = \frac{1}{1+2^2} = 1/5 = 0.2$
$x_3 = x_0 + 3h = 3$	$f(x_3) = 1/1+3^2 = 1/10 = 0.1$
$x_4 = x_0 + 4h = 4$	$f(x_4) = 1/1+4^2 = 1/17 = 0.0588$
$x_5 = x_0 + 5h = 5$	$f(x_5) = 1/1+5^2 = 1/26 = 0.0385$
$x_n = x_6 = x_0 + 6h = 6$	$f(x_6) = 1/1+6^2 = 1/37 = 0.027$

(i) By Trapezoidal Rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\left(f(x_1) + f(x_2) + \cdots + f(x_n)\right)\right]$$

$$= \frac{1}{2}\left[(1+0.027) + 2(0.5+0.2+0.1+0.0588+0.0385)\right]$$

$$= \frac{1}{2}\left[2.8216\right]$$

$$= 1.4108$$

(ii) By Simpson's Rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(f(x_0)+f(x_6)) + 4(f(x_1)+f(x_3)+f(x_5)) + 2(f(x_2)+f(x_4))\right]$$

$$= \frac{1}{3}\left[(1+0.027) + 4(0.5+0.1+0.0385) + 2(0.2+0.0588)\right]$$

$$= \frac{1}{3}[4.0986]$$

$$= 1.3662.$$

Ques: (i) The value of $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's Rule

Ques: Find the approximate value of

$$I = \int_0^1 \frac{dx}{1+x}$$

using; (i) Trapezoidal Rule (ii) Simpson's Rule