Mid point Method

$$y_{j+1} = y_{j-1} + 2hf_j \quad , \quad j = 0, 1, \ldots, N-1 \qquad \text{— (1)}$$

put, $j = 1$

$$y_2 = y_0 + 2hf_1 \quad , \quad y_0 \text{- Given}$$

$j = 2$

$$y_3 = y_1 + 2hf_2 \quad , \text{ Here } y_1 \text{ - unknown}$$

The value of y_1 is unknown, we have to find the value of y_1 by Taylor's series method of second order

$$y_1 = y(a) + hy'(a) + \frac{h^2}{2} y''(a) \qquad \text{— (2)}$$

a- initial point.

Ques:
Apply mid-point method to solve the initial value problem

$$\frac{dy}{dx} = yx^3 - 1.5y = f(x, y)$$

from $x = 0$ to 2 where $y(0) = 1$ by using $h = 1$.

Soln: $y_0 = 1$ (Given), $h = 1$

Firstly, we have to find y_1

$$y_1 = y(0) + hy'(0) + \frac{h^2}{2} y''(0)$$

$y(0) = 1$,

$$y' = yx^3 - 1.5y$$

$$y'(0) = y(0)^3 - 1.5 \times 1$$

$$= -1.5$$

$$y'' = 3x^2y + x^3\frac{dy}{dx} - 1.5\frac{dy}{dx}$$

$$y''(0) = 3 \times 0 \times 1 + 0 \times (-1.5) - 1.5 \times (-1.5)$$

$$= 2.25$$

$$y_1 = 1 + 1 \times (-1.5) + \frac{1}{2} \times (2.25)$$

$$= 1 - 1.5 + \frac{2.25}{2}$$

$$= \frac{-1.0 + 2.25}{2}$$

$$\Rightarrow \frac{1.25}{2} = 0.625$$

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$$y_2 = y_0 + 2hf(x_1, y_1) \qquad \left[x_1 = x_0 + h = 0 + 1 = 1\right]$$

$$= 1 + 2\left[y_1 x_1^3 - 1.5 y_1\right]$$

$$= 1 + 2\left[0.625 \times 1 - 1.5 \times 0.625\right]$$

$$= 1 + 2\left[0.625 - 0.9375\right]$$

$$= 1 + 2(-0.3125)$$

$$= 1 - 0.625$$

$$= 0.375$$

$$y_3 = y_1 + 2hf(x_2, y_2) \qquad \left[x_2 = x_0 + 2h = 2\right]$$

$$= 0.625 + 2\left[y_2 x_2^3 - 1.5 y_2\right]$$