Page No.
Date

Numerical Method (Sec I)

Intermediate Value thm

If $f(x)$ is continuous on $[a, b]$ and $f(a) \cdot f(b) < 0$, then the eq^n $f(x) = 0$ has at least one real root or an odd no. of roots in $[a, b]$

Bisection Method

Note : It is based on intermediate value thm

Algorithm :

Step 1.) Find the interval $[x_0, x_1]$ such that $f(x_0) \cdot f(x_1) < 0$

Step 2.) Find $x_2 = \frac{1}{2}(x_0 + x_1)$

Step 3.) If $f(x_2) = 0$, then 'x_2' is desired root. then stop

Step 4.) If $f(x_2) \neq 0$, then check the sign of $f(x_0) f(x_2)$

If $f(x_0) f(x_2) < 0$, then we take x_2 as our new x_1 and repeat the above process.

Note : If $f(x_2) \neq 0$, then check the sign of $f(x_0) \cdot f(x_2)$
If $f(x_0) \cdot f(x_2) < 0$ then $x_2 \rightarrow x_1$
If $f(x_0) \cdot f(x_2) > 0$ then $x_2 \rightarrow x_0$
the repeat the above procedure.

Date

Eg: Perform five iteration of the bisection method to obtain the smallest +ve root of the eqn
$$f(x) = x^3 - 5x + 1 = 0$$

Soln:

let $x_0 = 0$, $x_1 = 1$.

Since, $f(0) > 0$ & $f(1) < 1$, the smallest +ve root lies in the interval $(0, 1)$

Now, seqn of interval are given below.

x_0	x_1	x_2	$f(x_0)$	$f(x_2)$	$f(x_0)f(x_2)$
0	1	0.5	1	-1.37500	-ve
0	0.5	0.25	1	-0.23938	-ve
0	0.25	0.125	1	0.37695	-ve
0.125	0.25	0.1875	0.37695	0.06909	+ve
0.1875	0.25	0.21875	0.6909	-0.08328	+ve

Step 1. $x_0 = 0$, $x_1 = 1$ $f(x_0) \cdot f(x_1) < 0$

step 2.) $x_2 = \frac{0+1}{2} = \frac{1}{2} = 0.5$.

Step 3) $f(x_0) = 1$, $f(x_2) = -1.3750$

Since, $f(x_0) \cdot f(x_2) = -1.3750 < 0$

then replace x_1 as x_2.

Step 1.) $x_0 = 0$, $x_1 = 0.5$

step 2.) $x_2 = \frac{0+0.5}{2} = 0.25$

Date

Step 3 $f(x_0) = 1$, $f(x_2) = f(0.25) = -.23938$

$f(x_0) \cdot f(x_2) < 0$

then again replace x_1 as x_2.

Now, $x_0 = 0$, $x_1 = 0.25$

$$x_2 = \frac{0 + 0.25}{2} = 0.125$$

$f(x_0) = 1$, $f(x_2) = f(0.125) = 0.37695$

$f(x_0) \cdot f(x_2) > 0$

then replace x_0 as x_2

i.e $x_0 = 0.125$, $x_1 = 0.25$

$$x_2 = \frac{0.125 + 0.25}{2} = 0.1875.$$

Ques: Perform five iteration of the bisection Method to obtain the smallest +ve root of the eq^n

$$f(x) = x^3 - 3x + 1$$

Hint: $x_0 = -2$, $x_1 = -1$