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Solid State Physics
Superconductivity and Magnetism

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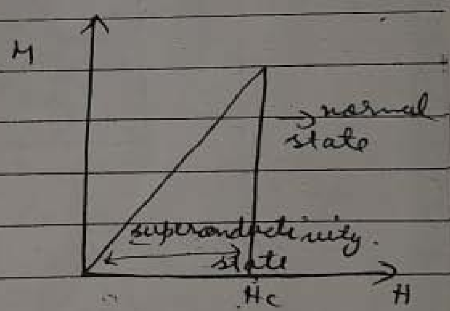
Type I and type II superconductors

superconductors can be classified in two categories.

1 Type I
2 Type II

Type I are superconductors which ~~exist~~ shows or exhibit complete meissner effect i.e show perfect diamagnetism and are also known as soft superconductors.
In this case diamagnetism abruptly disappears at critical magnetic field value H_c and transition from superconducting to normal state is sharp. Pure specimen of many materials exist this behaviour
eg Al, Zn, Hg, etc.

Type - I



Type II
They do not have sharp transition but a range of field called initial field H_{c1} at which flux ejection starts meaning a transition towards

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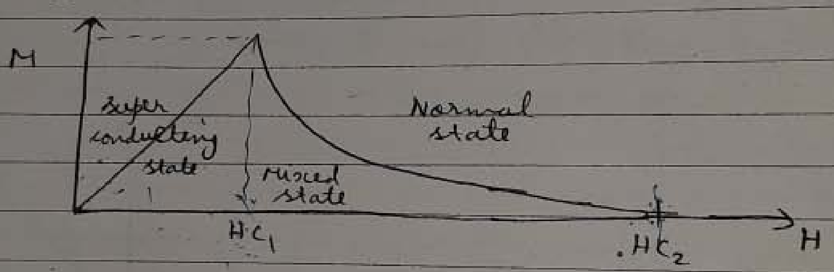
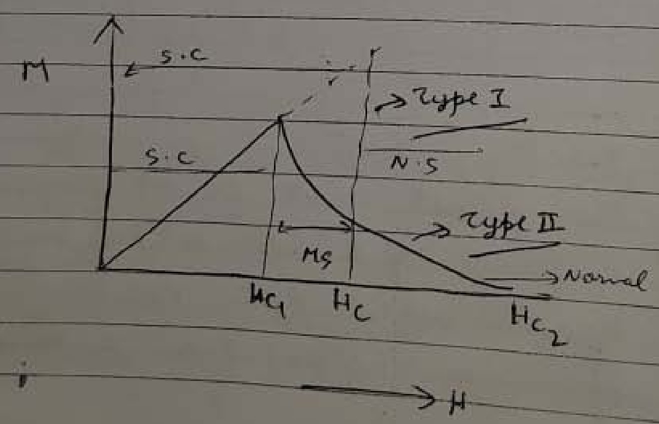
superconducting state and transition is complete at HC_1 where $HC_1 < HC_2$ - In b/w HC_1 and HC_2 specimen show mixed behaviour where there is small amount of magnetic field present in specimen. Below HC_1 specimen is completely in superconducting state and above HC_2 it is in normal state. generally HC_2 is 100 times more than HC_1. They are also called hard superconductors.
eg Ta, V, Nb

Type II superconductor with high HC_2 can be used to made high superconducting magnet. Advantage is light weight and negligible dessipation of energy.





6 marks (meissner + type I + type II) diagrams

Meissner effect.

Acc to Meissner effect, the superconductor expels magnetic flux completely, by measuring the magnetic field in neighbourhood of specimen in different cases. It can be established that as the temp. is lowered to T_c the specimen becomes superconducting and flux is pushed out for all the temp $T < T_c$. It was also demonstrated that effect is reversible i.e when temp is raised above T_c, flux suddenly starts to penetrate in the specimen and it returns back to normal state.

In this state, the magnetic induction inside the specimen is given by.

$$B = \mu_0 (H + M)$$

where $H \rightarrow$ external applied field

$M \rightarrow$ Magnetisation produced inside specimen

$$B = 0 \quad \rightarrow \frac{M}{H} = -1$$

susceptiblity $\rightarrow \chi = -1 \rightarrow$ complete diamagnetic

$E = J\rho \qquad E = J\rho$

for superconductor $\rho = 0$

$$\underset{0}{\nabla \times E} = -\frac{\partial B}{\partial t}$$

$B =$ constant $\rightarrow$ But it is zero inside specimen so maxwell law does not follow

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derivation
3 marks

one time
LE + PD
(10 marks)

London equation

Since Meissner effect and field penetration in superconductors could not be proved by maxwell electrodynamics equations alone, london brothers F. London, H. London examined the magnetic aspect qualitatively and showed it is necessary to include two addition equations to explain the effect completely. They used two fluid model. Acc to this model, a superconductor is supposed to be composed of two distinct types of electrons i.e normal e^-'s and super e^-'s. Normal e^-'s are usual electrons but super electrons behave in a very different way because they have zero entropy experience no scatterring and have long coherence length of the order of 10^4 Å. At $T = T_c$, all e^-s are normal but as T is lowered, an increasing proportion become super e^-'s. At $T \approx 0K$ they all become super e^-. So London brothers put forward the idea that total sum of conduction e^- $= n = n_s + n_n$ where

n_s = super e^-

n_n = normal e^-

n = total conduction e^-

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It was also assumed that normal current and super current flow parallel and since super current flow without resistance so it will carry entire current induced by small transity electric field and hence normal e^- will remain quite inert in the process and will therefore be ignored in discussion.

Let us suppose transient electric field that arise b/w superconductors that freely accelerates super e^- without dessipation. If V_s is average velocity, m is mass and e^- is charge of super e, then eq of motion of super e^- can be written as

$$m\frac{dv_s}{dt} = -e \cdot E \quad \text{—} ①$$

and the current density of super electron is

$$J_s = -e n_s v_s \quad \text{—} ②$$

from eq ① and ②, we get

London first equation
$$\boxed{\frac{dJ_s}{dt} = \frac{n_s e^2}{m} E} \quad \text{—} ③$$

Above eq show that if $E = 0$, J is finite and constant ~~and vice versa~~.

But we know that $J = \sigma E$ (eq of normal current density

i.e if $E = 0, J = 0$

but for superconductor if $E = 0$

$J =$ constant (from eq. 3).

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if we take curl of J_s then we get i.e curl of eq. 3

$$\nabla \times \left(\frac{dJ_s}{dt}\right) = \frac{n_s e^2}{m}(\nabla \times E)$$

$$\rightarrow \nabla \times \left(\frac{dJ_s}{dt}\right) = \frac{n_s e^2}{m}\left(\frac{-dB}{dt}\right)$$

or $$\boxed{\nabla \times J_s = -\frac{n_s e^2 B}{m}}$$ — 4 London II equation

using

$$\nabla \times E = -\frac{dB}{dt}.$$

3 marks

London Penetration depth.

London brothers successfully explained Meissner effect by adding two new eqs to four maxwell eqs., but it was noticed in some conductors, the flux does not drop to zero at surface but decreases exponentially, i.e flux varies with depth of specimen, which was also proved by London brothers.

$$\nabla \times B = \mu_0 J_s \quad \text{— (1)}$$

taking curl of above eq.

$$\nabla \times \nabla \times B = \nabla \times (\mu_0 J_s) \quad \text{— (2)}$$

$\nabla \cdot B = 0$ II Maxwell eq

$$\nabla(\nabla \cdot B) - \nabla^2 B = \mu_0 (\nabla \times J_s) \quad \text{— (3)}$$

$$-\nabla^2 B = \mu_0 (\nabla \times J_s) \quad \text{— (4)}$$

using LondonII eq in eq we get

$$\nabla^2 B = \frac{\mu_0 n_s e^2}{m} B \quad \text{— (5)}$$

$$\nabla^2 B = \frac{B}{\lambda^2} \quad \text{— (6)}$$

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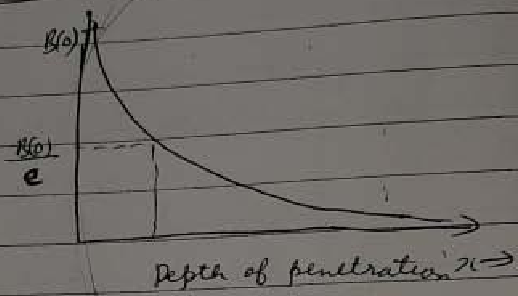
when $\lambda = \left(\frac{m}{\mu_0 n_s e^2}\right)^{1/2}$ is called penetration depth

One dimensional form of eq (6) will be

$$\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{\lambda^2} B_z \quad \text{--- (7)}$$

$$\boxed{B_z(x) = B_z(0) \exp\left(-\frac{x}{\lambda}\right)} \quad \text{--- (8)}$$



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Penetration depth has been verified and measured in different no of cases and found to be in agreement with the theoretical value of around 500 A°. Penetration depth is also found to depend upon temperature acc. to relation.

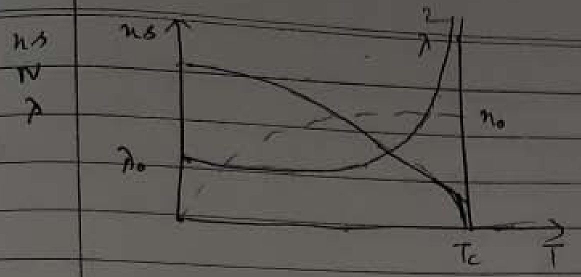
$$\boxed{\lambda(T) = \lambda(0)\left[1 - \frac{T^4}{T_c^4}\right]^{-1/2}}$$

Acc. to above relation λ increases with increase of T and become ∞ at $T = T_c$ which is expected because at $T = T_c$, the substance changes from super-conducting state to normal state and field can penetrate into whole specimen and it has infinite depth of penetration.

Further, since london penetration depth and no of superelectrons n_s is inversely related to each other and is also temp. dependent. We can also find out a relation b/w n_s and T.

$$\boxed{n_s = n_0\left(1 - \frac{T^4}{T_c^4}\right)}$$

Both relations can be graphically expressed as



We can also define a parameter W called order parameter given by.

$$W = \frac{n_s}{n_0} = \left(1 - \frac{T^4}{T_c^4}\right)$$

which characterizes the degree of order in superconducting state

BCS theory.

The modern theory of superconductivity was put forward by Bardeen, Cooper & Schrieffer in 1957 and hence named as BCS theory. The theory successfully explained all the observable effect such as zero resistivity, meissner effect and isotope effect etc.
Important aspects of theory are as follows.

1 Electron - Electron interaction via lattice deformation.

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Unit - 3

Magnetism

Magnetism is a phenomenon because of which the magnetic material or non magnetic material experiences a force due to presence of magnet. Materials which are attracted or repelled by magnet are called magnetic material.
eg Fe, Co, Ni, etc
& materials which do not respond to magnet are non-magnetic material
eg rubber, wood, leather, feather, etc.
Magnetic fields can be uniform, non-uniform and symmetric similar to electric field except magnetic monopole do not exist like we have existence of isolated +ve or -ve charges.
Unit of Magnetic field are N/Am or Tesla
1 Tesla = 10^{-4} gauss

Magnetic flux density,
The term magnetic induction B or Magnetic field strength H are used to describe the magnetic field
H represent magnetic field outside specimen & B is field that passes through specimen.
B → magnetic flux density.
$B = \mu_0 H$
μ_0 = Permeability of free space
$= 4\pi \times 10^{-7}$ Henry/m

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Intensity of Magnetization (M)
when a specimen is placed in a magnetic field it gets magnetized. Intensity of magnetization 'M' is magnetic moment per unit volume.

$$M = \frac{\mu m}{\Delta V}$$

$$B = \mu_0 H + \mu_0 M$$
$$= \mu_0 (H + M)$$

Susceptiblity. χ
It is defined as ratio of Magnetisation by magnetic field strength.

$$\chi_m = \frac{M}{H}$$

The ease with which magnetic material can be magnetised.

Permeability.

$$B = \mu_0 (H + M)$$
$$= \mu_0 (H + \chi H)$$
$$= \mu_0 (1 + \chi) H$$

But $B = \mu_0 \mu_r H = \mu H$

so $\mu H = \mu_0 (1 + \chi) H$

or $\boxed{\mu = \mu_0 (1 + \chi)}$

$$\mu_0 \mu_r = (1 + \chi) \mu_0$$
$$\boxed{\mu_r = 1 + \chi} \quad — (A)$$

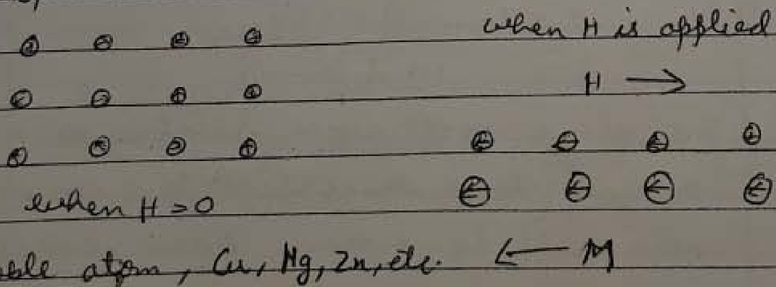
The above eq (A) gives relationship b/w permeability & susceptiblity. conceptually permeability means how easily a material would allow magnetic field lines to pass through it,

Classification of Magnetic material

I Diamagnetic materials.

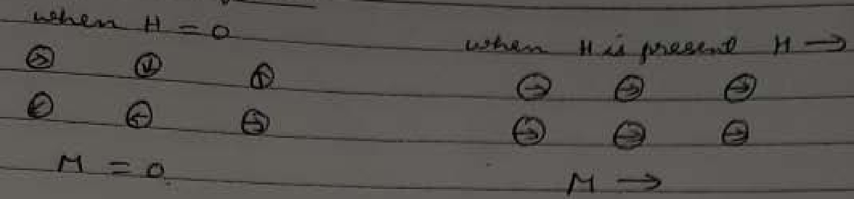
They are those which opposes magnetisation i.e when external field H is applied. the orientation of electronic orbitals in atoms are such that vector sum of magnetic moments are zero.
The alignment of magnetic moments happens in presence of external magnetic field but in the direction opposite to the applied field.
Diamagnetic materials are independent of temp.
relative permeability < 1
and they tend to move from stronger to weaker magnetic field in presence of non uniform magnetic field.

Superconductors exibt this behaviour. i.e field lines are ejected out of specimen.



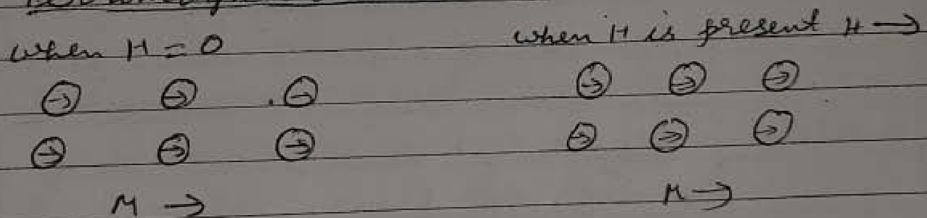
→ all noble atom, Cu, Hg, Zn, etc.

In paramagnetic



Paramagnetism is observed in materials in which orbitals are not completely filled or unpair spin exist.
In this case, magnetic moments are randomly orientated, so net magnetization is almost zero when magnetic field is not applied (H = 0)
However on application of H, the moments align ~~so~~ in direction of field giving rise to magnetization.
Both susceptiblity & permeability > 1
eg → Al, Cr, etc

Ferromagnetic materials.



In ferromagnetic materials, magnetic moment is due to unpair spin and aligned in particular direction in absence of magnetic field. and material exhist permanent magnetic moments. These are furthur

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enhanced by coupling interaction b/w magnetic moment of adjacent atom so that they all tend to align in same direction. (which is absent in paramagnetic material)

Maximum possible magnetization of these materials is referred to as saturation magnetisation.
Ferromagnetic materials have large +ve value of susceptiblity and they are temp. dependent and above a critical temp. i.e Curie Temp, material becomes paramagnetic.

<u>Antiferromagnetic materials</u>
In these materials, the magnetic moment, coupling in individual atom does not always align constructively as in ferromagnet. The alignment of spin moments of adjacent atom is in opposite direction. Thus net moment is zero.
They have small +ve value of susceptibility
eg MnO

<u>Ferrimagnetic Materials.</u>
They are in some sense similar to ferromagnets & also to antiferromagnets.
They have net magnetic moment as ferromagnent; but at same time, net

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magnetic moment is not as large as if all magnetic atoms are coupled constructively. Thus in ferrimagnetic substances, some of magnetically active atoms couple constructively and an unequal no also couple distructively.
Ferrimagnets are not electrically conductive but are used in high frequency application, eg - microwave devices, phase shifters, etc.
eg Fe_3O_4, $ZnFe_2O_4$ etc.

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Sources of Magnetic Moments

(1) Magnetic moment or the three factors which are source of permanent magnetic moments are:
1) Orbital Magnetic moment of e^-
2) Spin magnetic moment of e^-
3) Spin magnetic moment of nucleus.

1) Orbital magnetic moment of e^-
Acc. to classical atomic theory of magnetism, the magnetic moment can be written in terms of angular momentum $\vec{L}$, mass of e^-, charge of e^- as

$$\vec{\mu}_m = \frac{-e\vec{L}}{2m}$$

but acc. to quantum theory;

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$L = m_l \hbar$ (m_l = orbital magnetic quantum no.)

L is quantised given as

$\vec{L} = m_l \hbar$

$\vec{L} = \frac{m_l h}{2\pi}$

expression for magnetic moment

$\vec{\mu}_m = -\frac{eL}{2m}$

$= -\frac{e m_l \hbar}{2m}$

$= \frac{-e m_l h}{-2m \, 2\pi}$

$$\boxed{\begin{array}{l} \mu_m = -m_l \mu_B \\ \text{where } \mu_B = \frac{eh}{4\pi m} \end{array}} \rightarrow \text{Bohr Magneton}$$

μ_B is called Bohr Magneton whose value is 9.27×10^{-24} Am^2 and is quantum of orbital magnetic moment and is taken as 1 unit of magnetic moments of atomic system.

The total orbital magnetic moment is determined by sum of magnetic moments of individual e- within allowed rules (Hund's rule & Pauli's exclusion principal) and magnetic moments of complety filled shell is zero, thus only atoms with partial filled shells will have non zero orbital magnetic moment.

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2 Spin magnetic moment of e^-
Spin angular momentum is given by
$\mu_s = \frac{-eS}{2m} = \frac{-em_s\hbar}{2m}$

$= -m_s \mu_B$

$\mu_B = \frac{e\hbar}{2m_e} \rightarrow \frac{h}{2\pi} = \frac{eh}{4\pi m}$

3 Spin magnetic moment due to nucleus.
The contribution to magnetic moment comes from nuclear spin. Just like Bohr magneton μ_B is defined in case of e^-, nuclear magneton can be defined for nucleus due to nuclear spin which can be written as $\mu_n = \frac{eh}{4\pi M_n}$

$M_n >> m_e$

$\mu_n << \mu_e$

$\mu_n = 5.27 \times 10^{-27} Am^2$

Langevin's theory of diamagnetism
The theory was proposed by Paul Langevin in 1905 and main contribution of this theory is to reveal the -ve magnetism which is present in diamagnetic materials on application of applied field even though the material does not have magnetic moment of its own. The theory can be explained by considering a e^- of mass m, circulating in orbit of radius r, in absence of external field, the magnitude