Statistical Mechanics

by

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Class- B.Sc. (H) Physics, **Section-2**

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Department-Physics and Electronics

Unit:-

Classical Theory of Radiation

Topic- 1. Properties of Thermal Radiation

Radiation : The process by which heat is transferred directly from one body to another without affecting the intervening medium, is called "radiation." It is by radiation that the heat from the sun reaches the earth.

Radiant Energy : To explain heat-transfer by radiation, it is universally assumed that all bodies at all times are emitting energy. This energy is called "radiant energy" or "thermal radiation" and is in the form of electromagnetic waves. These waves travel with the velocity of light and are transmitted through vacuum or through a medium like air. When they fall on a body which is not transparent to them, they are absorbed and their energy is converted into heat.

The thermal radiation emitted by a body, per unit time and per unit surface area, depends on the nature of its surface and on its temperature. At low temperatures the rate of emission is small. As the temperature is increased, the rate of emission increases rapidly in proportion to the fourth power of the absolute temperature. Futher, the thermal radiation emitted by a body is a mixture of waves of different wavelengths. At ordinary and moderately high temperatures mostly the longer waves (infra-red) are emitted, but at very high temperatures shorter waves are also emitted.

Properties of Thermal Radiation : The thermal radiation is completely identical to light.

(i) It travels through empty space with the same speed as light.

(ii) It travels in straight lines in the same sense as light.

(iii) It obeys the inverse-square law.

(iv) It undergoes reflection, refraction and total internal reflection obeying the same laws as light.

(v) It exhibits interference, diffraction and polarisation.

(vi) It exerts a small, but finite, pressure on the surface on which it is incident.

Topic 2. Temperature dependence

Prevost's Theory of Exchanges : Prevost, in 1792, enunciated a "theory of exchanges" to account for the phenomenon of radiation. According to this theory, *every body is continuously emitting radiant energy in all directions at a rate depending only on the nature of its surface and its temperature, and it is absorbing radiant energy from all surrounding bodies at a rate depending on its surface and the temperature of the surrounding bodies.*

- **The quality and quantity of radiation inside a uniform temperature enclosure is independent of nature of the wall of the enclosure and nature of bodies present in it**

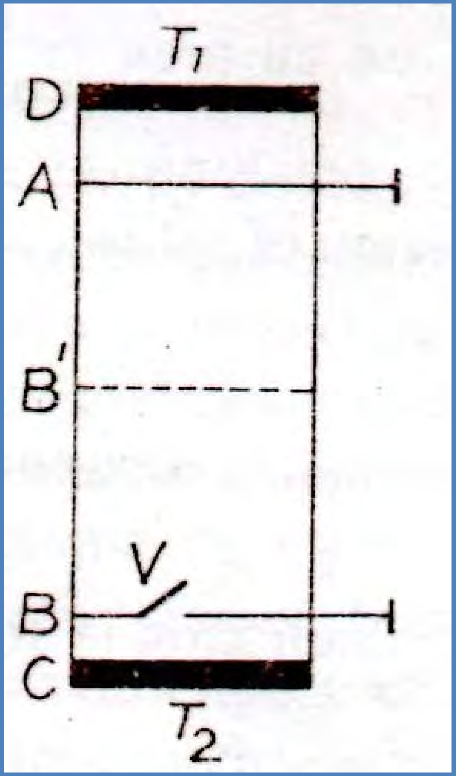
- **And depend only on the temperature of the enclosure.**

- **A body placed in the enclosure would acquire the temperature of that.**

Tropic 3. Pressure of Radiation

- Maxwell proved from electromagnetic theory that radiation should exert a pressure, and this is equal to energy density of radiation of parallel beam

- Bartoli's proof the existence of radiation pressure which as follow:



Figure

Description of Figure shown in previous slide

Let us 'imagine' a cylinder with perfectly reflecting walls, and its end-faces closed by perfectly conducting 'black' pieces D and C . Let D and C be maintained at constant temperatures T_1 and T_2 (by keeping them in contact with heat sources of infinite *capacity and temperatures* T_1 and T_2) ; T_1 *being greater than* T_2. Let A and B be two perfectly reflecting screens ; and V be a valve in the screen B .

Let the initial positions of the screens A and B be as shown, the valve V being open. The space AC is filled with radiation which is in equilibrium with black body C at temperature T_2 ; whereas the space AD is filled with radiation which is in equilibrium with black body D at temperature T_1 .

Let us now imagine that the valve V is closed and the screen B is pushed up to the position B' . The radiation, initially in AB , is compressed within the space AB' and its density in AB' becomes greater than that in AD . If now the screen A is withdrawn, the space $B'D$ will have much more radiation than necessary for equilibrium with the black body D at temperature T_1 . The excess radiation will therefore be absorbed by D . Thus we have transferred heat from a colder body C to a hotter body D. This, by the second law of thermodynamics, could be done by the expenditure of the work. This means that we have done some work in pushing the screen B upward, that is, B has been pushed upwards against a pressure. This pressure can be due only to radiation.

Pressure of Diffuse Radiation:

Pressure of Diffuse Radiation : Let us consider radiation of energy-density u (amount of radiation per unit volume) falling normally upon a surface. Radiation travels with the speed of light c. Therefore, the radiation reaching unit surface area per second is

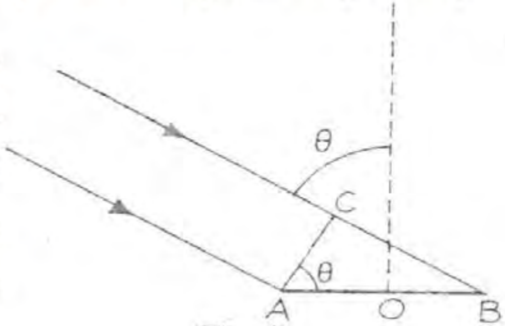
$$c \times u,$$

which is the amount of radiation contained in a cylinder of length c and cross-sectional area unity.

We know that with energy u is associated a momentum (u/c)* . Therefore, the momentum reaching per unit surface area per second is $c \times (u/c) = u$. Since the rate of change of momentum per unit surface area is the pressure, the surface absorbing the radiation falling upon it experiences a pressure given by

$$p = u.$$

Now, suppose the radiation is falling on a surface AB at an angle θ with the normal (Fig. 4). The energy contained in a cylinder of length c and cross-sectional area unity now falls on an area $\frac{1}{\cos\theta}\left[\because \frac{AB}{AC} = \frac{1}{\cos\theta}\right]$. The force per unit area on the surface in the direction of falling radiation is now



(Fig. 4)

$$\frac{\text{pressure}}{\text{area}} = \frac{u}{1/\cos\theta} = u\cos\theta.$$

Its normal component is

$$(u\cos\theta)\cos\theta = u\cos^2\theta.$$

If this radiation is absorbed, the pressure experienced by the surface is

$$p = u\cos^2\theta.$$

Let us now consider radiation in a uniformly-heated enclosure. This can be considered as equivalent to a large number of beams, say N, of equal intensity distributed uniformly in all directions. The pressure experienced by a surface AB in such an enclosure would be

$$p = \frac{1}{N}\, u\, \Sigma\cos^2\theta, \qquad \text{...(i)}$$

where $\Sigma\cos^2\theta$ is the sum of the values of $\cos^2\theta$ for all the beams.

To evaluate $\Sigma\cos^2\theta$, we draw a hemisphere of radius r around the centre O of the surface AB (Fig. 5). On this hemisphere we take a ring of radius

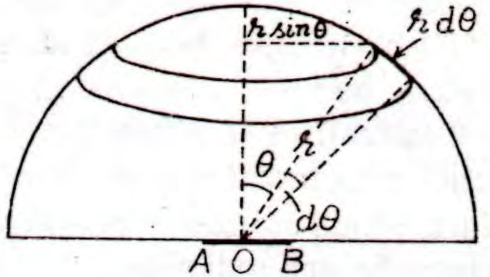
$r \sin\theta$ and width $r\,d\theta$, as cut by two cones of semi-angles θ and $\theta + d\theta$ drawn from O as apex. If dN is the number of beams crossing this ring, then

$$\frac{dN}{N} = \frac{\text{area of ring}}{\text{area of hemisphere}}$$

$$= \frac{2\pi (r \sin\theta)\, r\, d\theta}{2\pi r^2}$$

$$= \sin\theta \, d\theta$$

or $\quad dN = N \sin\theta \, d\theta$.



(Fig. 5)

The value of $\cos^2\theta$ is same for all the dN beams crossing the ring. Thus, for the ring, we have

$$\Sigma \cos^2\theta = \cos^2\theta \; dN = N \cos^2\theta \sin\theta \, d\theta$$

and so for the entire hemisphere, we have

$$\Sigma \cos^2\theta = \int \cos^2\theta \; dN = N \int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta^* = \frac{1}{3} N.$$

Substituting this value in eq. (i), we get

$$\boxed{p = \frac{u}{3}}.$$

Thus, the pressure due to diffuse radiation is one-third the energy-density.

(b) Analogy between Black-body Radiation and Perfect Gas : There is an analogy between black-body radiation, that is, radiation in an enclosure and perfect gas. A perfect gas is an assembly of molecules having all velocities from 0 to ∞ and moving in all directions. In the same way, radiation in an enclosure consists of all wavelengths and is proceeding in all directions. In a gas, the molecules have Maxwellian velocity distribution which shows marked resemblance with the energy distribution among the wave-lengths of the black-body radiation. The gas molecules and the radiation both exchange momentum with the walls of the enclosure and exert pressure on them.

Topic 4. Prefect Black Body

Perfectly Black Body : A perfectly black body is one which absorbs completely all the radiation, of whatever wave-length, incident on it. Since it neither reflects nor transmits any radiation, it appears black whatever the colour of the incident radiation may be.

Emissive and Absorptive power

Emissive Power : *The emissive power of a body, at a given temperature and for a given wave-length, is defined as the radiant energy emitted per second per unit surface area of the body per unit wave-length range*. Thus, if $e_\lambda \, d\lambda$ be the radiant energy between the wave-lengths λ and $\lambda + d\lambda$, emitted per second per unit surface area of a body of a temperature T, then in the limit $d\lambda \to 0$, e_λ will be the "emissive power" of the body at temperature T for the wave-length λ.

In fact, the above is the definition of the "monochromatic" or "spectral" emissive power e_λ. The total emissive power e (say) is the radiant energy emitted per second per unit surface area at *all* wave-lengths.

Absorptive Power : *The absorptive power of a body, at a given temperature and for a given wave-length, is defined as the ratio of the radiant energy absorbed per second by the surface of the body to the total energy falling per second on the same area*. Thus, if an amount dQ of radiant energy between wave-lengths λ and $\lambda + d\lambda$ be falling on a body at a temperature T, of which a fraction $a_\lambda \, dQ$ be absorbed, then, in the limit $d\lambda \to 0$, a_λ will be the "absorptive power" of the body at temperature T for the wave-length λ.

Again, the above is the definition of the "monochromatic" or "spectral" absorptive power a_λ. The total absorptive power a (say) is the ratio of the energy absorbed per second to the energy falling per second at *all* wave-lengths.

Topic 5. Kirchhoff's law

Kirchhoff's Law : It states that **the ratio of the emissive power to the absorptive power for radiation of a given wave-length is the same for all bodies at the same temperature , and is equal to the emissive power of a perfectly black body at that temperature .**

Proof : Let a body be placed in a uniformly-heated enclosure at a constant temperature. Let an amount of radiant energy dQ between the wave-lengths λ and $\lambda + d\lambda$ be incident per second on unit surface area of the body. Let a_λ be the absorptive power of the body for the wave-length λ . Then , an amount of energy $a_\lambda \, dQ$ will be absorbed per second by unit surface area of the body. The balance $(1 - a_\lambda) \, dQ$ of the incident energy will be reflected or transmitted.

Now, let e_λ be the emissive power of the body at wavelength λ . Then an amount of energy $e_\lambda \, d\lambda$ between the wavelengths λ and $\lambda + d\lambda$ will be emitted per second by unit surface area of the body by virtue of its temperature.

Thus, the total energy sent out by unit area of the body per second is $(1 - a_\lambda) \, dQ + e_\lambda \, d\lambda$.

Now, the presence of the body does not effect the quantity or quality of the radiation stream in the enclosure. Therefore, the energy sent out by unit area of the body per second should be equal to the energy received, and hence

$$(1 - a_\lambda) \, dQ + e_\lambda \, d\lambda = dQ$$

or

$$e_\lambda \, d\lambda = a_\lambda \, dQ . \qquad \text{...(i)}$$

Now, for a perfectly black body the absorptive power $a_\lambda = 1$ (for all wavelengths). Therefore, if E_λ be the emissive power of a black body, we shall have

$$E_\lambda \, d\lambda = dQ \, .$$

Substituting this value of dQ in eq. (i), we get

$$e_\lambda \, d\lambda = a_\lambda \, E_\lambda \, d\lambda$$

or

$$\frac{e_\lambda}{a_\lambda} = E_\lambda \, .$$

Since E_λ is constant at a given temperature, it follows that

$$\frac{e_\lambda}{a_\lambda} = \text{constant} \, ,$$

for all substances at the same temperature. This is Kirchhoff's law.

- **Kirchhoff's law tells us that good absorbers are good emitters.**

- **If the body absorb radiation of particular wavelength strongly, its also emits the same radiation strongly.**


Thank you