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B.Sc (PS) Computer Science VI Sem
Numericals (Dielectrics)

Dielectrics
(Examples & Numericals)

Q1. Two parallel plates have equal and opposite charges and seprated by a ~~distance~~ dielectric 5mm thick having dielectric constant 3. If the field intensity in the dielectrics is 10^6 V/m. Calculate (a) the Polarisation P in the dielectric (b) the displacement D in the dielectric (c) the energy density in the dielectric.

Ans1. Field intensity $\vec{E}$ in the dielectric $= 10^6$ V/m

(a) Polarization vector

$$\vec{P} = \epsilon_0 (\epsilon_r - 1) \vec{E}$$

Where $\epsilon_r = 3$ & $\epsilon_0 = 8.85 \times 10^{-12}\, C^2/N.m^2$

$$P = 8.58 \times 10^{-12} \times (3-1) \times 10^6 = 17.76 \times 10^{-6}\, C/m^2$$

(b) Displacement vector $\vec{D} = \epsilon_r \epsilon_0 \vec{E}$

$$= 3 \times 8.85 \times 10^{-12} \times 10^6$$

or $\vec{D} = 26.55 \times 10^{-6}\, C/m^2$.

(c) Energy density $= \frac{1}{2} \epsilon_r \epsilon_0 \vec{E}^2 = \frac{1}{2} \times 3 \times 8.85 \times 10^{-12} \times 10^{12}$

$$= 13.82\, J/m^3.$$

Q2. An isotropic material of relative permittivity ϵ_r is placed normal to a uniform external electric field with an electric displacement vector of magnitude $5 \times 10^{-4}\, m^2$. If the volume of the slab is $0.5\, m^3$ and magnitude of polarization is $4 \times 10^{-4}\, m^2$. Find the value of ϵ_r and total dipole moment of the slab.

Ans2. In the Problem, given units of $\vec{P}$ and $\vec{D}$ are not in S.I. units, thus electric displacement vector is expressed as

$$D = E + P$$

Thus $E = D - P$

$D = 5\times10^{-4} m^2$, $P = 4\times10^{-4} m^2$, $\epsilon_r = ?$

V = volume of Dielectric = $0.5 m^3$.

$\therefore E = (5\times10^{-4} - 4\times10^{-4}) m^2$

$= 1\times10^{-4} m^2$

& $\epsilon_r = \frac{D}{E} = 5$.

$\vec{P} = \frac{\text{Total dipole moment}}{\text{volume}} = \frac{\vec{p}}{V}$

or $\vec{p} = \vec{P}V = 4\times10^{-4}\times0.5 = 2\times10^{-4} m^2$.

Q3- For a gas the value of dielectric constant at 0°c is 1.000038. Calculate the electric susceptibility at this temperature.

Ans3. The dielectric constant is related to electric susceptibility χ_e by

$\epsilon_r = 1 + \chi_e$

$\chi_e = \epsilon_r - 1$

$\chi_e = 1.000038 - 1 = .000038$

Q4- The dielectric constant of helium at 0°c and 1 atm pressure is 1.000074. Find the dipole moment induced in each helium atom when the gas is in electric field of intensity 100V/m.

Ans4- The Polarization per unit volume is the dipole moment.

$\vec{p} = \frac{\vec{P}}{V}$

If N is the number of atoms per unit volume then $\vec{p} = \frac{\vec{P}}{V}$

& $\vec{P} = \epsilon_0(\epsilon_r - 1)\vec{E}$

$\therefore \quad \vec{p} = \frac{\epsilon_0 (K-1) \vec{E}}{N}$

$= \frac{8.85 \times 10^{-12} \times .000074}{N} \times 100$

at NTP $\quad N = \frac{6 \times 10^{23}}{22.4 \times 10^{-3}}$.

$\therefore$ Induced dipole moment

$\vec{p} = \frac{8.85 \times 10^{-16} \times 74 \times 22.4 \times 10^{-3}}{6 \times 10^{23}}$

or $\hat{p} = 24.46 \times 10^{-40}$ C-m.

Q5. A parallel plate capacitor is filled with insulating material of dielectric constant ϵ_r, what effect does this have on its capacitance?

Ans5. As the field is confined to the space between the plates, the dielectric will reduce $\vec{E}$ and hence potential difference V will be reduced by a factor of $1/K$. Thus the capacitance

$C = q/V$.

is increased by a factor of the dielectric constant i.e

$C = \epsilon_r C_0$

where C_0 is the capacitance w/o dielectric.

Q6. For a dielectric the value of dielectric constant is 1.329, calculate its electric susceptibility (χ_e)?

Ans6. We know that $\epsilon_r = 1 + \chi_e$

$\chi_e = \epsilon_r - 1$

$\chi_e = 1.329 - 1 = 0.329$.

$\boxed{\chi_e = 0.329}$.

Q7. Show that the optical refractive index 'n' is related to the electronic dielectric constant ϵ_r and explain how the ionic and electronic polarizability can be separated from measurement of dielectric constants and the refractive index of the material.

Ans7. From the field theory, we know that

$$c = v\sqrt{\mu_r \epsilon_r}$$

Where v is the phase velocity of the e.m wave in the given dielectric medium, c is the velocity of light, ϵ_r is the relative dielectric constant and μ_r is the relative permeability. The refractive index 'n' is given by the ratio of vel. of light in vaccum to the vel. of light in the medium.

$$n = c/v = \sqrt{\mu_r \epsilon_r}$$

for dielectrics $\mu_r = 1$.

So we get $n^2 = \epsilon_r$

In case of alkali halides the polarization is made up of electronic as well as ionic polarization. A d.c measurement of ϵ_r will give $N\alpha_e + N\alpha_i$ and we have Clausius-Mossotti relation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{(N\alpha_e + N\alpha_i)}{3\epsilon_0}$$

Since ionic contribution is neg. thus above eq^n. can be written as

$$\boxed{\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0}}$$

↓ Lorentz eq^n.

Knowing $n \rightarrow \alpha_e$, knowing d.c value of ϵ_r, α_i can also be calculated.

Q8. The static dielectric constant of water is 8.1 and its index of refraction is 1.33. What is the % contribution of ionic polarizability?

Ans.8. We know that

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N(\alpha_e + \alpha_i)}{3\epsilon_0}$$

At optical frequencies know if α_i very small and $\epsilon_r = n^2$ then above eq^n becomes

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha_e}{3\epsilon_0}$$

$$\text{or} \quad \frac{n^2+2}{n^2-1} \times \frac{\epsilon_r + 2}{\epsilon_r - 1} = \frac{\alpha_e}{\alpha_e + \alpha_i}$$

$$\text{but} \quad 1 - \frac{\alpha_e}{\alpha_e + \alpha_i} = \frac{\alpha_i}{\alpha_e + \alpha_i}$$

∴ % contribution of ionic polarizability is

$$\frac{\alpha_i}{\alpha_e + \alpha_i} \times 100 = \left(1 - \frac{n^2 - 1}{n^2 + 2} \cdot \frac{\epsilon_r + 2}{\epsilon_r - 1}\right) \times 100$$

$$= \left(1 - \frac{(1.33)^2 - 1}{(1.33)^2 + 2} \times \frac{8.1 + 2}{8.1 - 1}\right) \times 100$$

$$= 70\%$$