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B.Sc PS Computer Science VI Sem
Dielectrics

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Thus $n = \sqrt{\epsilon_r}$, where ϵ_r is the relative permittivity. Using this value of ϵ_r in equation (3).

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha}{3\epsilon_0} \qquad (6).$$

This is called ~~lorentz~~ lorenz-lorentz formula.

Behaviour of Dielectrics in Alternating field

When a dielectric ~~is~~ material is placed in an alternating field the orientation of the dipoles and hence the polarization will tend to reverse when the polarity of the field changes. As long as the frequency remains low ($< 10^6$ Hz) there is no significant lag in polarization with alterations of the field. The permittivity is independent of freq. and has same magnitude as in the static field. When the freq. is increased, the dipoles will not be able to rotate rapidly and their oscillators will lag behind those of the field. With further inc. in freq., the permanent dipoles in the medium will be unable to follow the field and contribution to static permittivity from this molecular process i.e the orientation polarization stops. This usually happens in the radio frequency range ($10^6 - 10^{11}$ Hz). ~~At~~

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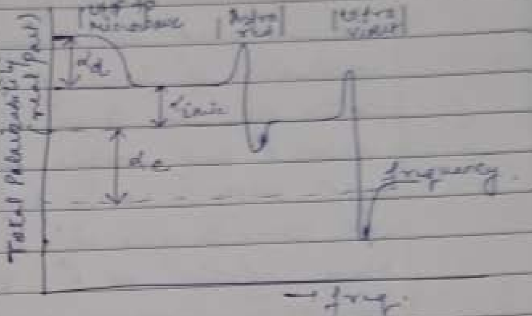
$(10^{11}-10^{14}$ Hz$)$, the heavy positive and negative ions can not follow the field alterations and contribution to permittivity from atomic or ionic polarization stops and only electronic polarization remains.

Thus permittivity of the dielectric material decreases with increase in freq. and this phenomenon is called anomalous dielectric dispersion.

Dielectric absorption: Dispersion arising during the transition from full atomic polarization at radio frequencies to negligible atomic polarization at optical freq. is called dielectric absorption.



Freq. dependence of various polarization

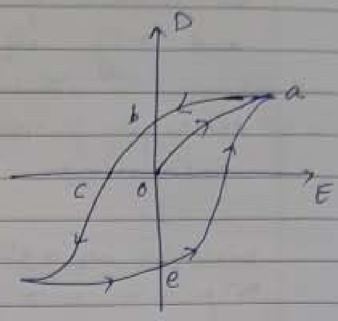
Dielectric Relaxation: – Dispersion arising during the transition from full orientational polarization at zero or low freq. to negligible orientational polarization at high freq. is called dielectric relaxation.

Polarization: – Polarization is greatly affected by the frequency of applied voltage. Electronic polarization occurs at all freq. ranging upto optical freq. $(\sim 10^{15}$ Hz$)$. Ionic polarization occurs upto infrared range $(\sim 10^{13}$ Hz$)$ and orientation polarization occurs when the freq. of applied voltage is in the audio range $(\sim 10^{11}$ Hz$)$.

I.T.O

Dielectric losses:-

When a dielectric is placed in an alternating field there is an electrical energy loss in the dielectric which is converted into heat energy. This process is known as loss of power and is used in industrial applications of heating wood, plastic etc. The polarization vector $\vec{P}$ and electric displacement vector $\vec{D}$ varies periodically with time. In general $\vec{P}$ & $\vec{D}$ may lag behind in phase relative to $\vec{E}$. The phenomenon of displacement vector $\vec{D}$ lagging behind electric field vector $\vec{E}$ is termed as hysteresis. This phenomenon is similar to the hysteresis in magnetism where the magnetic flux $\vec{B}$ lags behind magnetic field intensity $\vec{H}$. Thus in the present case the curve between $\vec{E}$ & $\vec{D}$ is called hysteresis curve, and area under the curve measured the loss of energy called the dielectric loss.



Curve between D & E.

It can be explained as follows:
let the electric field vector be given as

$$\vec{E} = \vec{E}_0 \cos\omega t$$

so that $\vec{D} = \vec{D}_0 \cos(\omega t - \delta)$ —— (1)

$$= \vec{D}_0 \cos\delta \cos\omega t + \vec{D}_0 \sin\delta \sin\omega t$$

$$= \vec{D}_1 \cos\omega t + \vec{D}_2 \sin\omega t$$

where $\vec{D}_1 = \vec{D}_0 \cos\delta$, $\vec{D}_2 = \vec{D}_0 \sin\delta$ and δ is the phase. For most dielectrics $\vec{D}_0$ is proportional to $\vec{E}_0$ but ratio $(\vec{D}_0/\vec{E}_0)$ is frequency dependent. ~~we~~ To shown, we introduce two frequency dependent dielectric constant as

$$\epsilon_r' = \left|\frac{\vec{D}_1}{\vec{E}_0}\right| = \left|\frac{\vec{D}_0}{\vec{E}_0}\right| \cos\delta$$

$$\epsilon_r'' = \left|\frac{\vec{D}_2}{\vec{E}_0}\right| = \left|\frac{\vec{D}_0}{\vec{E}_0}\right| \sin\delta$$

These two constants can be combined in single complex dielectric constant

$$\epsilon_r^* = \epsilon_r' + i\,\epsilon_r'' \quad \text{---------- (2)}$$

and $\vec{D} = \epsilon_r^* \vec{E}_0 e^{i\omega t} = \epsilon_r^* \vec{E}_0 (\cos\omega t + i\sin\omega t)$ -(3)

also $\tan\delta = \epsilon_r''/\epsilon_r'$ as ϵ_r' and ϵ_r'' are freq. dependent, the phase angle δ is also freq. dependent.

To show:- Energy dissipated in the dielectric :

Energy dissipated per second in the dielectric is

$$W = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \vec{I}\,\vec{E}\,dt$$

Putting the values of $\vec{I}$ and $\vec{E}$, we get

$$W = \frac{\omega}{2\pi}\left[-\omega\int_0^{2\pi/\omega} \vec{E}_0\vec{D}_1 \sin\omega t\cos\omega t\,dt + \int_0^{2\pi/\omega} \vec{E}_0\vec{D}_2\cos^2\omega t\,dt\right]$$

[where $\vec{I} = \omega(-\vec{D}_1\sin\omega t + \vec{D}_2\cos\omega t)$].

That ~~energy dissipated per second~~ term the ~~dielectric is~~

the value of the first integral is zero, thus

$$W = \frac{\omega}{2}\vec{D}_2\vec{E}_0$$

or $W = \frac{\omega}{2}\vec{D}_0\vec{E}_0\sin\delta$

But $\epsilon_r'' = \left|\frac{\vec{D}_0}{\vec{E}_0}\right|\sin\delta$

$$\therefore\ W = \frac{\omega}{2}E_0^2\,\epsilon_r''$$

Thus energy losses are proportional to ϵ_r'' or $\sin\delta$. Thus $\sin\delta$ is called loss factor and δ is the loss angle. For small value of δ, $\tan\delta$ is called loss factor as $\tan\delta = \sin\delta = \delta$. The dielectric loss at low frequencies is mainly due to d.c resistivity but at high frequencies the dielectric loss is mostly due to dipole rotations.

Classical theory of Electronic Polarizability:-

The dependence of electronic polarizability of frequency can be shown assuming the system as simple harmonic oscillator. The equation of motion in the local electric field $\vec{E}_{loc}\sin\omega t$ is

$$m\frac{d^2x}{dt^2} + m\omega_0^2 x = -e\vec{E}_{loc}\sin\omega t$$

putting $x = x_0\sin\omega t$, we get

$$m(-\omega^2+\omega_0^2)x_0 = -e\vec{E}_{loc}$$

or $$-x_0 = \frac{e\vec{E}_{loc}}{m(\omega_0^2-\omega^2)}$$

and the corresponding dipole moment has the amplitude

$$p_0 = -ex_0 = -\frac{e^2\vec{E}_{loc}}{m(\omega_0^2-\omega^2)}$$

which gives the electronic polarizability as

$$\alpha_e\,(\text{electronic}) = \frac{p_0}{E_{loc}} \frac{e^2/m}{(\omega_0^2-\omega^2)} \quad \text{---}(1)$$

showing its dependence on ω. ~~frequency dependence of various polarizabilities is~~

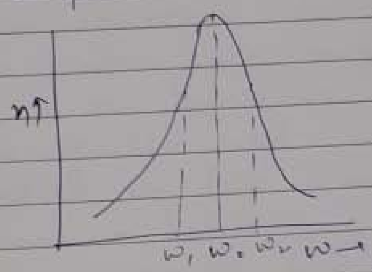
If there are Z electrons per atom and N atoms per unit volume, then the resulting electric susceptibility is

$$\chi_e = \frac{Nze^2/m\epsilon_0}{(\omega_0^2-\omega^2)} \quad \text{--- (2)}$$

and the index of refraction is given by

$$n^2 = 1+\chi_e = 1+\frac{Nze^2/\epsilon_0 m}{(\omega_0^2-\omega^2)} \quad \text{--- (3)}$$

The plot between n vs. ω shows a stray dispersion at the resonance freq. $\omega=\omega_0$. Equation (3) is called the dispersion relation.

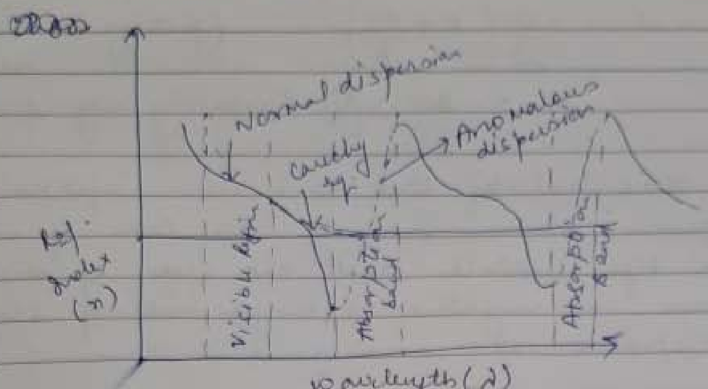


Normal and Anomalous Dispersion:-

When the refractive index of the medium varies with wavelength λ (i.e freq.), the phenomenon is called dispersion, and the medium is called dispersive medium. Mathematically represented by $dn/d\lambda$.

In general the refractive index (n) decreases with wavelength. The effect is called normal dispersion. But over small wavelength ranges there is often increase of refractive index due to an increased absorption of the radiation passing through the medium. This effect is known as anomalous dispersion. Figure shows graphically normal and anomalous dispersion.

Dielectrics Continued.



Normal and Anomalous dispersion

Characteristics of normal dispersion:-

(i) The refractive index decreases as wavelength increases.

(ii) The rate of increases of refractive Index with wavelength i.e $dn/d\lambda$, (i.e slope of curve) is greater at shorter wavelength. In other words dispersion increases as the wavelength decreases.

(iii) The normal dispersion may be explained by cauchy's empirical equation expression refractive index as a fn. of wavelength given by

$$n^2 = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} \quad (1)$$

where A, B & C are constants and ↓ rapidly as we proceed to H.O.T.

Retaining only first two terms

$$n^2 = A + \frac{B}{\lambda^2} \quad (2)$$

Differentiating above eq^n.

$$2n\frac{dn}{d\lambda} = -\frac{2B}{\lambda^3} \Rightarrow \frac{dn}{d\lambda} = -\frac{2B}{\lambda^3}\cdot\frac{1}{2n} \quad (3)$$

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$$= -\frac{B}{\lambda^3}\left(A + \frac{B}{\lambda^2}\right)^{-1/2}$$ (putting the value of n)

$$= -\frac{AB}{\lambda^3} \quad \text{--- (4)}$$ (retaining only first term)

This indicates that to a close approx. the dispersion varies inversely as cube of wavelength. -ve sign indicates that the slope of dispersion curve is negative & magnitude of slope i.e dispersion decreases as λ increases.

Sellmeier's Formula:-

The first dispersion formula of general application was given by Sellmeier in 1871, on the basis of elastic solid theory of light. He assumed that the medium contains elastically bound particles capable of vibrating with a natural frequency of vibration say ν_0 in the absence of any periodic force and if the radiation of this frequency is passed through the medium, the particles resonate and the energy is absorbed. For radiation of other frequencies the particles execute forced oscillations, the amplitude increasing as frequency of the radiation approaches the resonant frequency. These vibrations cause a change in the velocity of radiation. If all the elastically bound particles in the medium have a natural frequency corresponding to radiation of wavelength λ_0 in a vaccum, then Sellmeier's theory gives

$$n^2 = 1 + \frac{A\lambda^2}{\lambda^2 - \lambda_0^2} \quad \text{--- (5)}$$

A = Constt.