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B.Sc (PS) Comp. Sci VI Sem

Solid State Physics

Ferromagnetism

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Ferromagnetism:-

Like Paramagnetism, Ferromagnetism is also associated with the presence of permanent magnetic dipoles, but unlike paramagnetism, the magnetic moments of adjacent atoms in this case are aligned in a particular direction even in the absence of the applied magnetic field. Thus a ferromagnetic material exhibits a magnetic moment in the absence of a magnetic field. The magnetization existing in a ferromagnetic material in the absence of an ~~magnetic field~~ applied magnetic field is called the spontaneous magnetization. It exists below a certain critical temperature called the Curie Temperature T_c .

The alignment of magnetic moments below the Curie temperature is due to the exchange interaction between the magnetic ions. Above the Curie temperature, the thermal effects offset the spin alignment and the ferromagnetic substance becomes paramagnetic.

The ferromagnetic substances acquire a large magnetization in the presence of even a weak external magnetic field. They possess a large and positive value of susceptibility which in general, is not constant but varies with field strength. The variation of magnetization with field strength exhibits the well-known hysteresis curve. The μ_r of ferromagnetic materials are the elements such as Fe, Ni, Co, Gd & Dy, and a no. of alloys and oxides such as $MnBi$, $MnAs$, CrO_2 etc.

$$\text{So } B_E = dM.$$

Weiss theory of ferromagnetism:-

Theory of ferromagnetism put forward by Weiss is centered about two hypothesis:-

(i) A specimen of ferromagnetic material contains a number of small regions called domains which are spontaneously magnetized.

The magnitude of spontaneous magnetization of the specimen as a whole is determined by the vector sum of the magnetic moments of individual domains.

(ii) The spontaneous magnetization of each domain is due to the presence of an exchange field B_E , which tends to produce a parallel alignment of the atomic dipoles. The field B_E is assumed to be proportional to the magnetization M of each domain i.e.

$$B_E = dM \quad \text{--- (1)}$$

Where d is called Weiss-field constant and is independent of temperature.

Thus effective field on an atom or ion becomes-

$$B_{\text{eff}} = b + B_E = b + dM \quad \text{--- (2)}$$

Quantum theory of ferromagnetism:-

Consider a ferromagnetic solid containing N atoms per unit volume each having a total angular momentum quantum number J . By analogy we can write the expression for magnetization as

Co whereas for Mn and Cr it does not favor ferromagnetism. These elements like Mn & Cr do not exhibit ferromagnetism.

For the expression for exchange field B_E :

Assuming J_e to be constant for all the neighbouring pairs and considering contributions from the nearest neighbours only, we may write the exchange energy of an i^{th} atom as

$$U_i = -2zJ_e \sum_j S_i \cdot S_j \quad \text{--- (1)}$$

When the summation is over the nearest neighbours of the i^{th} atom, the instantaneous values of the neighbouring spins were replaced by their time averages. For z nearest neighbours eq. (1) can be written as

$$U_i = -2zJ_e (S_{zi} \langle S_{zj} \rangle + S_{yi} \langle S_{yj} \rangle + S_{xi} \langle S_{xj} \rangle) \quad \text{--- (2)}$$

For magnetization, M , along the z -axis, we have

$$\langle S_{xj} \rangle = \langle S_{yj} \rangle = 0 \quad \& \quad \langle S_{zj} \rangle = \frac{M}{g\mu_B N} \quad \text{--- (3)}$$

$$U_i = \frac{-2zJ_e S_{zi} M}{g\mu_B N} \quad \text{--- (4)}$$

It may be written as

$$U_i = -g S_{zi} \mu_B B_E \quad \text{--- (5)}$$

with $B_E = \frac{2zJ_e}{g^2 \mu_B^2} M$ --- (6)

Constant = A

assumed to be the same for any pair of atoms. Its value depends on the overlap of the charge distributions of the two atoms i.e. on the interatomic distance. In general, J_e is positive for large interatomic distances and negative for smaller ones. The expression (A) is known as the Heisenberg model of exchange energy. It also follows from (A) that if J_e is positive, the parallel arrangements of spins exhibit lower energy and hence is more stable as compared to the antiparallel arrangements, thereby producing magnetization. In a similar way, it can be concluded, that the negative value of J_e does not favour magnetism.

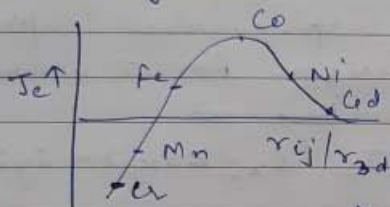


fig. 11

Plot of exchange integral J_e vs ratio of interatomic separation to the radius of the 3d orbit (r_{ij}/r_{3d})

The exchange integral for iron group of atoms is found to be positive in spite of the fact that the interatomic distance for this group is not large. This follows from the observations of Bohr and Slater which indicate that J_e is positive if the ratio r_{ij}/r_{3d} is greater than 3 but is not much larger than 3, r_{ij} being the distance between the atoms i and j and r_{3d} is the radius of the unfilled 3d-shell. The variation of J_e as a function of r_{ij}/r_{3d} is shown in fig. 11. It can be seen that ratio r_{ij}/r_{3d} favours ferromagnetism for Fe, Co, Ni and

Nature and origin of Weiss Molecular field Exchange interactions

The Weiss theory of ferromagnetism is based on the concept of ferromagnetic domains which are spontaneously magnetized due to the presence of an internal molecular field called the Weiss field or the exchange field B_E . The theory however, does not explain the origin and nature of this field.

It was Heisenberg who first proposed in 1926 that the Weiss field was the consequence of the quantum-mechanical exchange interaction between the atoms. This interaction arises due to the Pauli exclusion principle according to which any change in the relative orientation of the two spins would disturb the spatial distribution of charge, thus producing interaction between the two atoms.

Apparently the strength of exchange interaction between the adjacent atoms depends on the extent of overlap of their wave functions as well as the relative orientation of the electron spins but not on the spin magnetic moments. Thus this is an electrostatic and non-magnetic type of interaction.

Using Heitler-London theory of chemical bonding, it can be shown that the total energy of a system of two atoms contains an exchange energy term given by

$$U_{ij} = -2J_e S_i S_j \quad \text{--- (A)}$$

where S_i and S_j represent the spins of the two atoms and J_e is the exchange integral which is

$$M = NgJ\mu_B B_J(x) \quad \text{--- (3)}$$

where

$$B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right) \quad \text{(4)}$$

$$\text{and } x = \frac{gJ\mu_B B_{\text{eff}}}{kT} = \frac{gJ\mu_B}{kT} (B + \lambda M_s(T)) \quad \text{--- (5)}$$

$B \rightarrow$ applied field

where $M_s =$ spontaneous magnetization when $B=0$.

In case of spontaneous magnetization (when $B=0$) and eq (5) becomes

$$x = \frac{gJ\mu_B \lambda M_s(T)}{kT} \quad \text{--- (6)}$$

$$\therefore \text{or } M_s(T) = \frac{xkT}{\lambda gJ\mu_B} \quad \text{--- (7)}$$

as $T \rightarrow 0$ or $x \rightarrow \infty$, $B_J(x) \rightarrow 1$; the magnetic moments align themselves parallel to the field and the magnetization M becomes the saturation magnetization $M_s(0)$ or $M(0)$. Thus from eq. (3) gives

$$M_s(0) = NgJ\mu_B \quad \text{--- (8)}$$

from eq. (7) & (8) we obtain

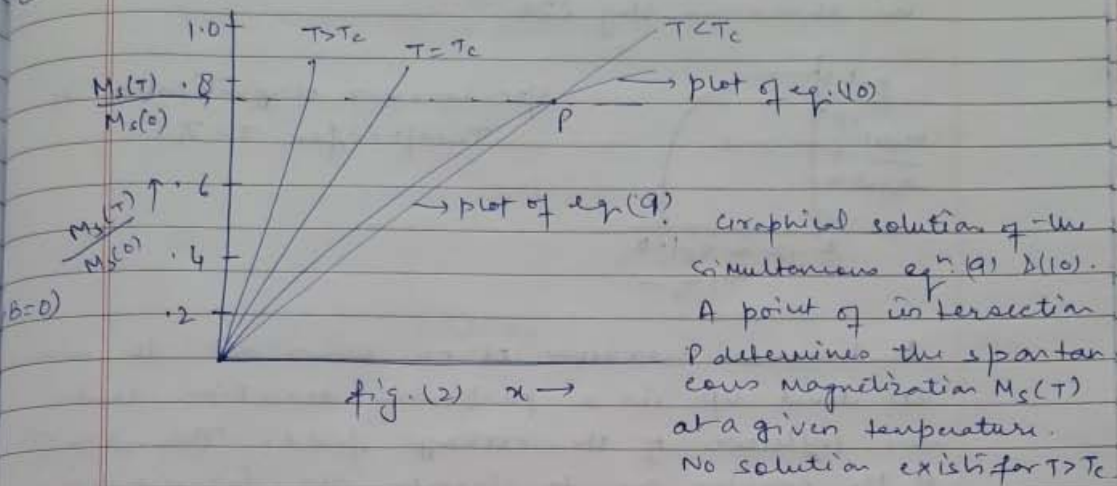
$$\frac{M_s(T)}{M_s(0)} = \frac{xkT}{\lambda Ng^2 J^2 \mu_B^2} \quad \text{--- (9)}$$

from eq. (3) we can write down

$$\frac{M_s(T)}{M_s(0)} = B_J(x) \quad \text{--- (10)}$$

$$\text{hence } B_J(x) = \frac{xkT}{\lambda Ng^2 J^2 \mu_B^2} \quad \text{--- (11)}$$

This equation can be solved graphically by plotting Brillouin function as a fn. of x as shown in fig. 2.

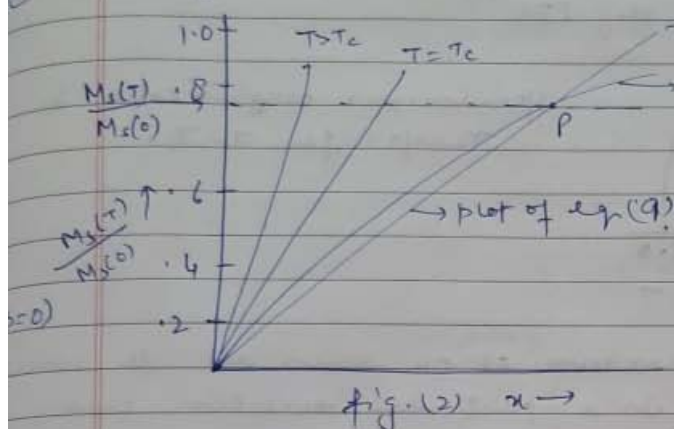


The Magnetization $M(T)$ at a given temperature can be obtained by solving eq. (9) & (10) simultaneously. The plot of $M(T)/M_s(0)$ vs x representing eq. (9) & (10) are shown in fig. (2). Note that the eq. (9) represents a straight line passing through the origin and having slope proportional to T .

At temp. equal to T_c (critical temp.), this line is tangent to the Brillouin ~~function~~ function at the origin. The intersection of the two plots at the point O represents a positive solution, but the magnetization corresponding to this point is unstable.

Another point of intersection appears for $T < T_c$ which indicates a non-zero value of M even for zero external field and hence corresponds to spontaneous magnetization. It also follows that the spontaneous magnetization decreases with the increase in temperature and vanishes

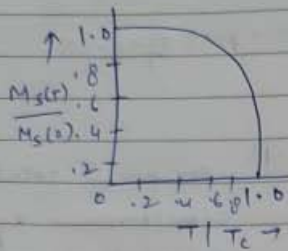
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Graphical solution of the simultaneous eq. (9) & (10). A point of intersection P determines the spontaneous magnetization $M_s(T)$ at a given temperature. No solution exists for $T > T_c$.

The Magnetization $M(T)$ at a given temperature can be obtained by solving eq. (9) & (10) simultaneously. The Plot of $M(T)/M_s(0)$ vs x representing eq. (9) & (10) are shown in fig. (2). Note that the eq. (9) represents a straight line passing through the origin and having slope proportional to T . At temp. equal to T_c (critical temp.), this line is tangent to the Brillouin ~~curve~~ function at the origin. The intersection of the two plots at the point O represents a positive solution, but the magnetization corresponding to this point is unstable. Another point of intersection appears for $T < T_c$ which indicates a non-zero value of M even for zero external field and hence corresponds to spontaneous magnetization. It also follows that the spontaneous magnetization decreases with the increase in temperature and vanishes

beyond the temperature T_c which is known as ferromagnetic Curie temperature. The variation of spontaneous magnetization with temperature is shown in fig. (3)



spontaneous magnetization vs Temp. for $T < T_c$.

It becomes maximum at 0K when all the moments are lined up in a particular direction under the influence of the exchange field. This variation of M_s enables one to classify the paramagnetic-ferromagnetic transition as a second order phase transition i.e. transition characterized by an order parameter (e.g. M_s in this case) which is non-zero only below T_c .

Paramagnetic Region:-

Consider the magnetization in the region well above the Curie temperature. For $T > T_c$, the spontaneous magnetization is zero and an external field will have to be applied to produce some magnetization. This field should, however, be weak enough to avoid the saturation state. In such a state, we find from eq. (5) that

$$B_T(x) = \frac{(J+1)x}{3J}$$

