

**19 March 2020**

**Dr. ANJU GUPTA**

B.Sc (PS) Computer Science VI Sem

Solid State Physics

Diamagnetism and Paramagnetism

magnetic moment is not as large as if all magnetic atoms are coupled constructively. Thus in ferrimagnetic substances some of magnetically active atoms couple constructively and an unequal no also couple destructively.

Ferrimagnets are not electrically conductive but are used in high frequency application, eg - microwave devices, phase shifters, etc.

eg  $\text{Fe}_3\text{O}_4$ ,  $\text{ZnFe}_2\text{O}_4$  etc.

22-9-2017

### Sources of Magnetic Moments

(4) Magnetic moment or the three factors which are source of permanent magnetic moments are:

1) Orbital magnetic moment of  $e^-$

2) Spin magnetic moment of  $e^-$

3) Spin magnetic moment of nucleus.

→ This is due to spin angular momentum of the nucleus

1) Orbital magnetic moment of  $e^-$

Acc. to classical atomic theory of magnetism, the magnetic moment can be written in terms of angular momentum  $\vec{L}$ , mass of  $e^-$ , charge of  $e^-$  as

$$\vec{\mu}_m = \frac{-e\vec{L}}{2m}$$

but acc. to quantum theory;

1) Magnetic moment due to the movement of electrons in orbit around the nucleus is due to orbital angular momentum. This is called orbital magnetic moment.

2) Magnetic moment due to spin of the electrons, i.e. due to spin angular momentum. This is called spin mag. mom.

$L = m_l \hbar$  ( $m_l \Rightarrow$  orbital magnetic quantum no.)  
 $L$  is quantised given as

$$\vec{L} = m_l \hbar$$

$$\vec{L} = \frac{m_l \hbar}{2\pi}$$

expression for magnetic moment

$$\vec{\mu}_m = -\frac{e\hbar}{2m} \vec{L}$$

$$= -\frac{e m_l \hbar}{2m}$$

$$= -\frac{e m_l \hbar}{2m} \frac{1}{2\pi}$$

$\mu_m = -m_l \mu_B$	$\rightarrow$ Bohr Magneton
where $\mu_B = \frac{e\hbar}{4\pi m}$	

$\mu_B$  is called Bohr Magneton whose value is  $9.27 \times 10^{-24} \text{ Am}^2$  and is quantum of orbital magnetic moment and is taken as unit of magnetic moments of atomic system.

The total orbital magnetic moment is determined by sum of magnetic moments of individual  $e^-$  within allowed rules (Hund's rule & Pauli's exclusion principle) and magnetic moment of completely filled shell is zero, thus only atoms with partially filled shells will have non zero orbital magnetic moment.

### Magnetic moment of simple atoms:-

The atomic magnetic moment is directly proportional to the total angular momentum of the atom. For the electrons in the  $s$ -orbital the orbital angular momentum is zero  $\rightarrow$  i.e. they do not contribute to orbital magnetic moment. For completely filled shells the net orbital magnetic moment is again 0. Atoms possess net orbital magnetic moment only if they have partially filled  $p, d$  or  $f$  orbitals. orbital mag. moment  $\rightarrow$  partially filled  $p, d, f$  orbitals  $\rightarrow$  spin mag. moment  $\rightarrow$  unpaired

Ex. 1) Hydrogen atom: - It has  $1e^-$  in s-orbital. It has zero orbital mag. moment but non-zero spin mag. moment. So its magnetic moment is equal to Bohr magneton.

2) Helium atom: - It has  $2e^-$  in 1s orbital. So zero orbital mag. moment & zero spin mag. moment due to paired e.

Spin magnetic moment of  $e^-$

Spin angular momentum is given by

$$\mu_s = \frac{-e\hbar}{2m} = -\frac{e\hbar}{2m} \sigma$$

$$= -\mu_B \sigma$$

$$\mu_B = \frac{e\hbar}{2m_e} = \frac{e\hbar}{4\pi m_e}$$

3) Spin magnetic moment due to nucleus.

The contribution to magnetic moment comes from nuclear spin. Just like Bohr magneton  $\mu_B$  is defined in case of  $e^-$ , nuclear magneton can be defined for nucleus due to nuclear spin which can be written as

$$\mu_n = \frac{e\hbar}{4\pi M_n}$$

$$M_n \gg m_e$$

$$\mu_n \ll \mu_B$$

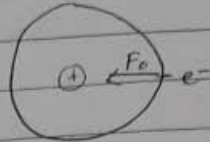
$$\mu_n = 5.27 \times 10^{-27} \text{ Am}^2$$

Langmuir's theory of diamagnetism

The theory was proposed by Paul Langmuir in 1905 and main contribution of this theory is to reveal the -ve magnetism which is present in diamagnetic materials on application of applied field even though the material does not have magnetic moment of its own. The theory can be explained by considering a  $e^-$  of mass  $m$ , circulating in orbit of radius  $r$ , in absence of external field, the magnitude

of centripetal force acting on  $e^-$  due to nucleus will be given by

$$F_0 = \frac{mv^2}{r}$$



$$F_0 = m\omega^2 r$$

where  $v = \omega r$

On application of external field  $B$ , Lorentz force will be acting

$$F = -e(\vec{v} \times \vec{B})$$

$$F = -eB\omega r$$

As a result net force will be given as

$$F_0 - eB\omega r = m\omega^2 r$$

$$\text{or } m\omega_0^2 r - eB\omega r = m\omega^2 r$$

$$\text{or } \omega^2 + \frac{eB\omega}{m} - \omega_0^2 = 0$$

$$\omega = \frac{-eB}{m} \pm \sqrt{\frac{e^2 B^2}{m^2} + 4\omega_0^2}$$

$$\omega = \frac{-eB}{2m} \pm \sqrt{\left(\frac{eB}{2m}\right)^2 + \omega_0^2}$$

$$\text{If } \omega_0^2 \gg \left(\frac{eB}{2m}\right)^2$$

$$\omega = \frac{-eB}{2m} + \omega_0$$

$$\omega = -\omega_L + \omega_0 \quad \text{--- (A)}$$

$$\omega_L = \frac{eB}{2m}$$

$\rightarrow$  Larmor's precessional frequency.

first  
The ~~second~~ term in eq A corresponds to change in frequency and is called,

Larmor's precessional frequency. The  $\pm$  sign of  $\omega_L$  implies that those  $e^-$  whose orbital moments were parallel to field are slowed down and those whose moments were antiparallel are speeded up by same amount. This change in frequency is due to application of applied magnetic field which causes magnetization.

$\rightarrow$  This result is called Larmor's theorem.  
In absence of external magnetic field, the current of circulating  $e^-$  are in random direction and cancel each other but in presence of magnetic field the current induce due to change in frequency of  $e^-$  causes magnetization. This current in presence of field can be written as

we know

$$\begin{aligned}\omega_L &= 2\pi \Delta \nu \\ &= \frac{eB}{2m} \\ \Delta \nu &= \frac{eB}{4\pi m}\end{aligned}$$

the magnetic moment of  $e^-$  will be given by

$$\begin{aligned}\Delta \mu_m &= iA \\ A &= \text{area of orbit} \\ i &= -e \left( \frac{\omega_L}{2\pi} \right)\end{aligned}$$

$$\begin{aligned}i &= (\text{charge}) \times (\text{revolution per time}) \\ &= -e \frac{\omega_L}{2\pi} = \frac{e^2 B}{4\pi m}\end{aligned}$$

If  $Z$  is no of  $e^-$  then

$$i = -\frac{Ze^2 B}{4\pi m}$$

So  $\mu_{lm}$  becomes

$$\mu_{lm} = -\frac{Ze^2 B}{4\pi m} \pi \langle r^2 \rangle$$



where  $A = \pi \langle r^2 \rangle$

$\langle r^2 \rangle$  is mean square radius of the  $e^-$  orbit on a plane  $\perp$  to field axis.

(\*) Since the field is applied along  $z$ -axis so

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$$

But mean square distance of  $e^-$  from the nucleus written in terms of average radii

$$\text{i.e. } \langle r_0^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

but since distribution of charges around nucleus is spherically symmetric so

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$$

$$\therefore \text{so } \frac{\langle r_0^2 \rangle}{\langle r^2 \rangle} = \frac{3}{2}$$

$$\text{so } \langle r^2 \rangle = \frac{2}{3} \langle r_0^2 \rangle$$

$$\text{so } \mu_{lm} = -\frac{Ze^2 B}{2\pi m} \frac{\pi}{3} \langle r_0^2 \rangle$$

$$\mu_{lm} = -\frac{Ze^2 B}{6m} \langle r_0^2 \rangle$$

But  $M = N \mu_{lm}$  ( $M = \text{total magnetization}$ )

$$\text{so } M = -\frac{ZNe^2 B}{6m} \langle r_0^2 \rangle$$

$$M = -\frac{ZNe^2 \mu_0 H}{6m} \langle r_0^2 \rangle$$

$$\chi = \frac{|M|}{H} = -\frac{Ze^2 N \mu_0}{6m} \langle r_0^2 \rangle$$

$$\text{A } \chi = -\frac{Ze^2 N \mu_0}{6m} \langle r_0^2 \rangle$$

Salient contribution of Langevin's theory.

- 1) The susceptibility of diamagnetic material is always -ve
- 2) The susceptibility has more contribution from outer orbits than inner orbits.
- 3) The susceptibility is independent of temp.
- 4)  $\chi$  diamagnetic susceptibility is of order of  $10^{-6}$ .

Theory of Paramagnetism

Paramagnetism arises due to reorientation and alignment of already existing magnetic dipoles in external magnetic field. The origin of magnetic dipole in paramagnetic solid is due to orbital and spin motion

Paramagnetism of  $e^-$  and contributes towards paramagnetism. It is observed in

- i) Metals
- ii) Atoms & molecules possessing an odd no. of  $e^-$  if free
- iii) Rare earth and actinide elements.

But once field is removed to go back to random directions.

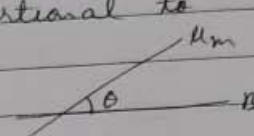
Langevin then proposed a classical theory for account of paramagnetic behavior & expression of paramagnetic susceptibility is obtained on basis of KTC. (Kinetic Theory of Gases)

The atoms and molecules possessing permanent magnetic moment to be treated as paramagnetic gases. They were assumed to be far apart from each other so that their mutual

interaction is totally neglected.

The paramagnetic gas particles have permanent magnetic moment  $\mu$ . In presence of field all permanent dipole align themselves in direction of field. The equilibrium distribution of dipole <sup>with reference to direction</sup> of field can be obtained using KTC as follows.

- 1) No of molecules that are orientated in a direction at angle  $\theta$  with external magnetic field is proportional to  $\sin \theta d\theta$



From idea of equipartition of energy,

the no of molecules having  $PE(E) = -\mu_m \cos \theta$  <sup>(Potential energy of each dipole)</sup> at absolute temp  $T$  is proportional to

$$\exp\left(-\frac{E}{k_B T}\right)$$

$$dN \propto \sin \theta d\theta$$

$$\propto \exp(-E/k_B T)$$

$$dN = C \exp\left(\frac{-E}{k_B T}\right) \sin \theta d\theta$$

$$dN = C \exp\left(\frac{\mu_m B \cos \theta}{k_B T}\right) \sin \theta d\theta \quad \text{--- (1)}$$

$$\text{let } \alpha = \frac{\mu_m B}{k_B T} \quad \text{--- (2)}$$

so eq (1) become

$$dN = C \exp(\alpha \cos \theta) \sin \theta d\theta \quad \text{--- (3)}$$

For all possible orientation from 0 to  $\pi$ , total no of molecules  $N$  will be

$$N = \pi \int_0^\pi C e^{\alpha \cos \theta} \sin \theta d\theta \quad (4)$$

$$\text{let } \cos \theta = x$$

$$-\sin \theta d\theta = dx$$

$$\text{or } N = \int_{-1}^1 C e^{\alpha x} dx = C \left[ \frac{e^{\alpha x}}{\alpha} \right]_{-1}^1$$

$$= \frac{C}{\alpha} [e^{\alpha} - e^{-\alpha}]$$

$$= C = \frac{N\alpha}{e^{\alpha} - e^{-\alpha}} \quad (5)$$

Now the intensity of Magnetisation will be derived as:-

Component of magnetic moment of molecule in the direction B is  $= \mu_m \cos \theta$  (6)

Total no of molecules with above orientation =  $dN$

so total magnetic moment because of  $dN$  molecules =  $\mu_m \cos \theta dN$  (7)

Total magnetic moment because of all molecules will be

$$M = \int_0^\pi \mu_m \cos \theta dN \quad (8)$$

Putting the expression of  $dN$  from eq (3)

$$M = \int_0^\pi (\mu_m \cos \theta) (C e^{\alpha \cos \theta} \sin \theta) d\theta \quad (9)$$

$$\text{let } \cos \theta = x$$

$$\sin \theta d\theta = -dx$$

$$M = C \mu_m \int_{-1}^1 x e^{\alpha x} dx \quad (10)$$

$$= \text{Cum} \left[ \frac{1}{\alpha} (e^{\alpha} + e^{-\alpha}) - \frac{1}{\alpha^2} (e^{\alpha} - e^{-\alpha}) \right]$$

using expression of  $c$  from eq (5)

$$= \frac{N\alpha}{e^{\alpha} - e^{-\alpha}} \text{Cum} \left[ \frac{1}{\alpha} (e^{\alpha} + e^{-\alpha}) - \frac{1}{\alpha^2} (e^{\alpha} - e^{-\alpha}) \right]$$

$$= \mu_m N \left[ \frac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}} - \frac{1}{\alpha} \right]$$

$$M = \mu_m N \left[ \coth \alpha - \frac{1}{\alpha} \right] \quad \text{--- (11)}$$

where  $\coth \alpha = \frac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}}$

also  $M = \mu_m N L(\alpha)$  --- (12)

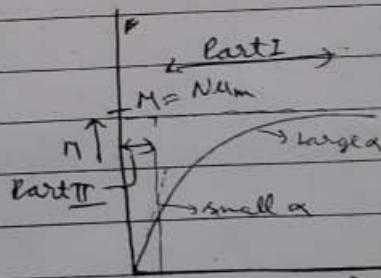
where  $L(\alpha) = \coth \alpha - \frac{1}{\alpha}$   
↓  
Langevin's function

$$M = M_s L(\alpha)$$

where  $M_s = \mu_m N$  ↓  
saturation magnetisation

$$M_s = M_s L(\alpha)$$

$$L(\alpha) = \frac{M}{M_s}$$



where  $\alpha = \frac{\mu_m B}{k_B T}$

→ α  
Langevin's curve.

The curve drawn b/w  $M$  and  $\alpha$  is called Langevin's curve. In this curve we can identify two regions Part I and Part II.

At low temp and large field,  $\alpha$  is very large ( $\alpha = \frac{\mu_m B}{k_B T}$ ), which is the condition at which all the dipoles are aligned in direction of field. This is case of saturation Magnetisation ( ~~$M = M_s$~~ )  $M = M_s$ . However the above situation is not practically possible due to temp effect and  $\alpha$  is generally small corresponding to lower portion of curve (portion II with linear slope) and it coincides with tangent of curve at origin.

from eq (1)  $M = N \mu_m \left( \coth \alpha - \frac{1}{\alpha} \right)$  (where  $M_s = N \mu_m$ )

Now expanding

$$\coth \alpha = \frac{1}{\alpha} + \frac{\alpha}{3} - \frac{\alpha^3}{45} + \dots$$

taking  $\alpha$  to be small,  $\therefore$  i.e. normal field strength + ordinary temp. then higher order terms can be neglected.

Retaining only first two terms

eq (1) becomes

$$M = N \mu_m \left( \frac{1}{\alpha} + \frac{\alpha}{3} - \frac{1}{\alpha} \right)$$

$$= N \mu_m \frac{\alpha}{3}$$

$$= \frac{N \mu_m}{3} \frac{\mu_m B}{k_B T}$$

$$M = \frac{N \mu_m^2 B}{3 k_B T}$$

curve is almost linear and coincides with

the tangent to the curve at the origin giving  $L(\alpha) = \frac{\alpha}{3}$ .

(13)

$$\text{Putting } B = \mu_0 H$$

$$M = \frac{N \mu_m^2 \mu_0 H}{3 k_B T}$$

$$\text{But } \chi = \frac{M}{H}$$

$$\chi = \frac{N \mu_m^2 \mu_0}{3 k_B T} \quad (14)$$

$$\chi = \frac{C}{T} \quad (15) \rightarrow \text{Curie law}$$

$$\text{when } C = \frac{N \mu_m^2 \mu_0}{3 k_B} \rightarrow \text{Curie constant}$$

Mass susceptibility

$$\chi_m = \frac{\chi}{\rho}$$

where  $\rho$  is density of substance,  $N_m$  is no of molecules per kg,  $N$  is no of molecules per  $m^3$  then mass of 1 molecule is  $\frac{1}{N_m}$  and volume of 1 molecule is  $\frac{1}{N}$ .

$$\text{then } \rho = \frac{N}{N_m}$$

$$\text{so mass susceptibility } \chi_m = \frac{\chi}{\rho}$$

$$\chi_m = \frac{\mu_m^2 N \mu_0}{3 k_B T \left( \frac{N}{N_m} \right)}$$

$$\chi_m = \frac{\mu_m^2 N_m \mu_0}{3 k_B T} \quad (16)$$

$$\chi_m = \frac{c}{T} \quad \text{--- (17) where } c = \frac{\mu_m^2 N_A \mu_0}{3k_B}$$

Molecular susceptibility.

No of molecules in a gram molecule is a avogadro no  $N_A$ . and molecular susceptibility can be written as

$$\chi_m = \frac{\mu_m^2 N_A \mu_0}{3k_B T}$$

$$= \frac{\mu_m^2 N_A^2 \mu_0}{3k_B N_A T}$$

$$\chi_m = \frac{\mu_0 (\mu_m N_A)^2}{3RT}$$

where  $R = k_B N_A =$  universal gas constant and  $\sigma = \mu_m N_A$  is defined as saturation value of magnetic moment per K mole

$$\text{so } \chi_m = \frac{\mu_0 \sigma^2}{3RT} \quad \text{--- (20)}$$

$$\chi_m = \frac{C_M}{T} \quad \text{--- (21)}$$

$$\text{where } C_M = \frac{\mu_0 \sigma^2}{3R}$$

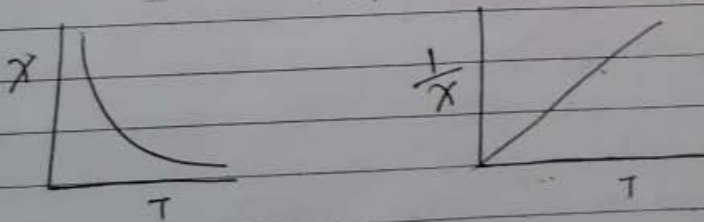


Fig (15), (17) and (21), i.e volume susceptibility, mass and molecular susceptibility of paramagnetic material all shows that

susceptibility is inversely proportional to  $T$  and is general independent of applied field.

The major limitation of this theory is in its assumption to ignore inter-molecular interaction which is not true in real cases. To include the interaction b/w magnetic dipole of atoms Weiss proposed a theory in which correction was made to field experienced by molecule. Further the paramagnetic susceptibility exhibited by certain solid salts, crystals, compressed & cooled gases was not ably accounted by this theory.

### Weiss theory of paramagnetism.

Weiss introduced a concept of effective field  $H_e$  experienced by molecules. In presence of molecular interaction they get influenced by magnetic field of each other and hence a molecular field is set b/w gas and it is proportional to  $M$ .

$$H_m \propto M$$

$$H_m = \lambda M$$

Effective field  $H_e = H + \lambda M$  — (22)

so the expression for magnetisation now becomes

$$M = \frac{\mu_m^2 N \mu_0}{3 k_B T} (H + \lambda M) \text{ — (23)}$$

Again using

$$\frac{\mu_m^2 NH}{3k_B T} \text{ for } M \text{ in above eq}$$

$$M = \frac{\mu_m^2 N \mu_0}{3k_B T} \left[ H + \frac{\mu_m^2 NH \lambda}{3k_B T} \right]$$

$$= \frac{\mu_m^2 NH \mu_0}{3k_B T} \left[ 1 + \frac{\mu_m^2 N \lambda}{3k_B T} \right]$$

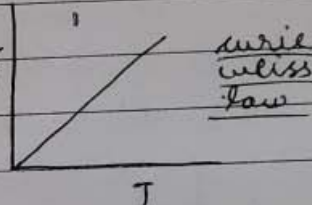
$$\chi = \frac{M}{H} = \frac{N \mu_0 \mu_m^2}{3k_B \left[ T - \frac{\mu_m^2 N \lambda}{3k_B} \right]}$$

$$= \frac{N \mu_0 \mu_m^2}{3k_B (T - T_c)} \quad \text{--- (24)}$$

where  $T_c = \frac{\mu_m^2 N \lambda}{3k_B} \rightarrow$  paramagnetic  
Curie point

$$\chi = \frac{C}{T - T_c} \quad \text{--- (25)}$$

$$\frac{1}{\chi}$$



10-2019

### Quantum theory of paramagnetism

In Langevin's classical theory of paramagnetism, the magnetic moments were assumed to be orientated at any angle  $\theta$  with reference to external magnetic field and hence susceptibility thus obtained did not have exact temp. dependence as found for some materials. However quantum theory considered by magnetic moments of

quantized, the magnetic dipole moment  $\mu$  and its component  $\mu_z$  in the direction of applied field cannot have arbitrary values. We know relation  $\mu \propto L$  and  $\mu \propto S$  magnetic dipole moment  $\mu$  of an atom and angular momentum

$$\mu = -g \mu_B J$$

atoms or ions like that in classical approach only but laid a restriction on their possible orientation, since the magnetic moments are

let if there are  $N$  no of atoms and  $J$  orbital and total spin momentum, total  $L$  &  $S$  are total angular momentum, total  $e^-$  in atom respectively. Then magnetic moment of each atom will be given by

$$\bar{\mu}_z = g \bar{J} \mu_B$$

where  $g$  is Lande's factor

$\mu_B$  is Bohr magneton

$\bar{J}$  is average angular momentum.

When an external magnetic field is applied, the axis of magnetic moment  $\bar{\mu}_z$  will precess about the direction of magnetic field and components of  $\bar{\mu}_z$  in direction of field can be written as

$$\begin{aligned} \text{Magnetic moment parallel to } \vec{H} \\ = M_s g \mu_B \vec{H} \quad m_J g \mu_B \vec{H} \end{aligned}$$

and the associated potential energy

$$= -M_s g \mu_B \mu_0 H$$

and acc. to Maxwell Boltzmann law, the ionic susceptibility,

$$\chi = \frac{1}{H} \left[ \frac{\sum_{M_s} M_s g \mu_B \exp(M_s g \mu_B \mu_0 H / k_B T)}{\sum_{M_s} \exp(M_s g \mu_B \mu_0 H / k_B T)} \right] \quad (1)$$

Case I.When  $M_S g \mu_B \mu_0 H \ll k_B T$ 

We will be retaining first two terms and neglecting H.O.T.

$$\exp\left(\frac{M_S g \mu_B \mu_0 H}{k_B T}\right) = 1 + \frac{M_S g \mu_B \mu_0 H}{k_B T}$$

$$\chi = \frac{1}{H} \left[ \frac{\sum_{M_S=-S}^S M_S g \mu_B \left[1 + \frac{M_S g \mu_B \mu_0 H}{k_B T}\right]}{\sum_{M_S=-S}^S \left(1 + \frac{M_S g \mu_B \mu_0 H}{k_B T}\right)} \right] \quad \text{--- (2)}$$

$$= \frac{1}{H} \left[ \frac{g \mu_B \sum_{M_S=-S}^S M_S + \frac{g^2 \mu_B^2 \mu_0 H}{k_B T} \sum_{M_S=-S}^S M_S^2}{\sum_{M_S=-S}^S 1 + \frac{g \mu_B \mu_0 H}{k_B T} \sum_{M_S=-S}^S M_S} \right] \quad \text{--- (3)}$$

We know that

$$\left. \begin{aligned} \sum_{M_S=-S}^S M_S = 0 \quad \text{and} \quad \sum_{M_S=-S}^S 1 = 2S+1 \end{aligned} \right\} \quad \text{(4)}$$

$$\text{and } \sum_{M_S=-S}^S M_S^2 = \frac{S(S+1)(2S+1)}{3}$$

Substituting in eq (3)

$$\chi = \frac{1}{H} \left[ \frac{0 + \frac{g^2 \mu_B^2 \mu_0 H}{k_B T} \cdot \frac{S(S+1)(2S+1)}{3}}{(2S+1) + 0} \right] \quad \text{--- (5)}$$

$$= \frac{g^2 \mu_B^2 \mu_0 S(S+1)}{3 k_B T} \quad \text{--- (6)}$$

$$\text{or } \chi = \frac{\mu_0 \mu_s^2}{3R_B T}$$

when  $\mu_s^2 = \mu_B^2 S(S+1)g$

so

$$\chi = \frac{\mu_0 \mu^2}{3R_B T}$$

$$\chi_p = \frac{\mu_0 \mu_B^2 g^2 I(I+1)}{3KT}$$

$$\chi_p = \frac{\mu_0 \mu_B^2 P_{eff}^2}{3KT} \quad \text{--- (A)}$$

Molecular susceptibility will be given by

$$\chi_m = \frac{N \mu_0 \mu^2}{3R_B T}$$

where  $P_{eff} = g \sqrt{I(I+1)}$

where  $P_{eff}$  is the effective number of Bohr magnetons and is given by

$$\chi_m = \frac{\mu_0 \mu^2 N^2}{3N R_B T}$$

where (A) is similar to the classical result.

$$= \frac{N^2 \mu^2 \mu_0^2}{3RT} \quad \text{--- (10)}$$

Also  $\mu_s = \mu$  can be written as

$$P_{eff} \mu_B \quad \text{where } P_{eff} = g \sqrt{S(S+1)} \quad \text{--- (11)} \rightarrow \text{effective no. of Bohr Magnetons}$$

for ground state

$$S = S - L$$

$$\text{so } P_{eff} = g \sqrt{S(S+1)} \quad \text{--- (12)}$$

using eq (11) and (12)  
 Equation (10) becomes

$$\chi = \frac{\mu_0 N P_{eff}^2 \mu_B^2}{3R_B T} = \frac{C}{T}$$

where  $\mu_s = P_{eff} \mu_B$

Case II

$$M_S g \mu_B \mu_0 H \gg k_B T$$

i.e. when  $T$  is low and field is high then

$$\text{let } x = \frac{g \mu_B \mu_0 H}{k_B T}, \text{ then}$$

$$M = N g \mu_B \sum_{M_S} M_S \exp(M_S x)$$

$$\sum_{M_S} \exp(M_S x)$$

$$= N g \mu_B \frac{d}{dx} \left[ \ln \sum_{M_S} \exp(M_S x) \right]$$

$$= N g \mu_B \frac{d}{dx} \left[ \ln (e^{Sx} + \dots + e^{-Sx}) \right]$$

$$= N g \mu_B \frac{d}{dx} \left[ \ln e^{Sx} [1 + e^{-2x} + \dots + e^{-2Sx}] \right]$$

$$= N g \mu_B \frac{d}{dx} \left[ \ln e^{Sx} \left( \frac{1 + e^{-(2S+1)x}}{1 - e^{-x}} \right) \right]$$

$$= N g \mu_B \frac{d}{dx} \left[ \ln \left( \frac{e^{Sx} - e^{-Sx}}{1 - e^{-x}} \right) \right]$$

Solving above term we get.

$$M = N g \mu_B S \left[ \frac{2S+1}{2S} \coth \left( \frac{(2S+1)a}{2S} \right) - \frac{1}{2S} \coth \left( \frac{a}{2S} \right) \right]$$

$$\text{where } a = x S$$

$B_S =$  Brillouin function

$$M = \frac{N g \mu_B S}{M_S} B_S(a)$$

$$M = M_S B_S(a) \quad \text{where } M_S = N g \mu_B S.$$

for  $x \gg 1$

$\coth a \approx 1$

$B_S(a) \approx 1$

which gives

for  $S = \infty$

$$\frac{M}{M_S} = \coth a = \frac{1}{a}$$

$$= L(a)$$

and for  $L = 1/2$

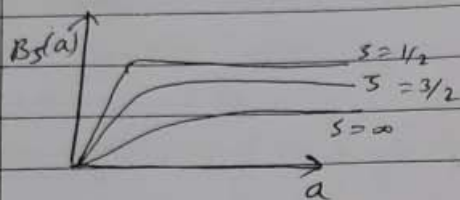
$$\frac{M}{M_S} = \tanh a = a - \frac{a^3}{3}$$

Thus the quantum results approach the

classical ones for all other values of  $T$

however the two

results differ considerably



### Ferromagnetism

The ferromagnetic materials show magnetism even in absence of external magnetic field. The phenomenon is known as spontaneous magnetization exhibited by transition metals and rare earth metals with half filled 3d and 4f shells.

This type of magnetism is associated with half filled nature of  $e^-$  in the shell. The net spin magnetic moment of ferromagnetic atoms is of same order as the magnetic moment of paramagnetic atoms under the influence of magnetic field but large magnetization is not due to the movement of individual atom but because of

