

Newton Raphson Method

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}} \quad f'(x_n) \neq 0$$

Algorithm

Step I.) Find out the point (x_0) which lies near the roots of eqⁿ $f(x) = 0$

Step II.) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} ; f'(x_0) \neq 0$

Step III.) If $f(x_1) = 0$, then we stop otherwise we replace x_0 by x_1 then go to step I.

Eg: Find x_1 and x_2 for eqⁿ $x^3 - 5x + 3 = 0$ by using Newton - Raphson Method.

Solⁿ:

Here x_0 is not given

$$f(x) = x^3 - 5x + 3$$

$$f(0) = 3, \quad f(1) = -1$$

$$\Rightarrow f(0) \cdot f(1) < 0$$

then root lies between $(0, 1)$

let $x_0 = 1$

$$f(x_0) = f(1) = 1^3 - 5(1) + 3 = -1$$

$$f'(x_0) = 3x_0^2 - 5$$
$$= 3 - 5 = -2$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$
$$= 1 - \frac{(-1)}{(-2)} = \frac{1}{2} = 0.5$$

Now, $f(x_1) = f(0.5) = (0.5)^3 - 5(0.5) + 3$

$$= 0.125 - 2.5 + 3$$
$$= 0.625$$

$$f'(x_1) = f'(0.5) = 3(0.5)^2 - 5$$
$$= -4.25$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 0.5 - \frac{0.625}{(-4.25)}$$
$$= 0.5 + 0.1471$$

$x_2 = 0.6471$

Ques: Find the iterative Methods based on the Newton Raphson Method for finding \sqrt{N} , $\frac{1}{N}$ and $N^{1/3}$ where N is the

real no. Apply the method to $N=18$. To obtain the results correct to two decimals.

Solⁿ

For \sqrt{N}

let $x = \sqrt{N}$

Squaring on both side

$$x^2 = N \Rightarrow x^2 - N = 0$$

$$f'(x) = 2x$$

By Newton Raphson method.

$$x_{k+1} = x_k - \frac{f_k}{f'_k}$$

$$x_{k+1} = x_k - \frac{(x_k^2 - N)}{2x_k}$$

$$= \frac{2x_k^2 - x_k^2 + N}{2x_k} = \frac{x_k^2 + N}{2x_k}$$

$$x_{k+1} = \frac{1}{2} \left(x_k + \frac{N}{x_k} \right) \quad \text{--- (1)}$$

We have to find for $N=18$, put in (1)

$$x_{k+1} = \frac{1}{2} \left(x_k + \frac{18}{x_k} \right)$$

Now, $x^2 - N = 0$

$$x^2 - 18 = 0$$

$$\Rightarrow x = 4.24$$

i.e. let $\boxed{x_1 = 4.24}$

$k=1$

$$x_{1+1} = x_2 = \frac{1}{2} \left(x_1 + \frac{18}{x_1} \right)$$

$$= \frac{1}{2} \left(4.24 + \frac{18}{4.24} \right)$$

$$= \frac{1}{2} (8.48)$$

$$\boxed{x_2 = 4.24}$$

$$\begin{aligned}x_3 &= \frac{1}{2} \left(x_2 + \frac{18}{x_2} \right) \\&= \frac{1}{2} \left(4.24 + \frac{18}{4.24} \right) \\&= \frac{1}{2} (8.48)\end{aligned}$$

$$\boxed{x_3 = 4.24}$$

for $\frac{1}{N}$ and $N^{1/3}$ do itself.