Dielectric Properties of Matter

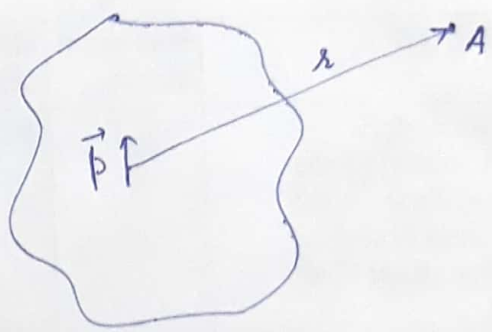
Polarisation

When a dielectric material made up of nonpolar molecules is placed in an electric field, the field will induce in each a tiny dipole moment pointing in the direction of same direction as the field. If the material is made up of polar molecules, each permanent dipole will experience a torque, tending to line it up along the field direction. Thus the material becomes polarized.

$$\text{Polarization } (\vec{P}) = \frac{\text{dipole moment}(\vec{p})}{\text{volume}}.$$

The Field of a polarized object

Bound charges



Electric potential at point A due to single dipole $\vec{p}$ is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$\vec{P} = \frac{\vec{p}}{d\tau'} \Rightarrow \vec{p} = \vec{P}\,d\tau$$

total potential is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{P}(\vec{r}\,')\cdot\hat{\imath}}{\imath^2}\, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int \vec{P}(\vec{r}\,')\cdot\frac{\hat{\imath}}{\imath^2}\, d\tau'$$

$$\nabla'\left(\frac{1}{\imath}\right) = \frac{\hat{\imath}}{\imath^2}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \vec{P}(\vec{r}\,')\cdot\nabla'\left(\frac{1}{\imath}\right) d\tau'$$

Rule.

$$\nabla\cdot(fA) = f(\nabla\cdot A) + A\cdot(\nabla f)$$

$$A\cdot(\nabla f) = \nabla\cdot(fA) - f(\nabla\cdot A)$$

$$\vec{P}\cdot\left(\nabla'\frac{1}{\imath}\right) = \nabla'\left(\frac{\vec{P}}{\imath}\right) - \frac{1}{\imath}(\nabla'\cdot\vec{P})$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \nabla'\cdot\left(\frac{\vec{P}}{\imath}\right) d\tau' - \frac{1}{4\pi\varepsilon_0}\int \frac{1}{\imath}(\nabla'\cdot\vec{P})\,d\tau'$$

$\Big\downarrow$ Acco. to Divergence theorem

$$= \frac{1}{4\pi\varepsilon_0}\oint \frac{\vec{P}}{\imath}\cdot d\vec{a}\,' - \frac{1}{4\pi\varepsilon_0}\int \frac{\nabla'\cdot\vec{P}}{\imath}\, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\oint \frac{\vec{P}\cdot\hat{n}}{\imath}\, da' - \frac{1}{4\pi\varepsilon_0}\int \frac{\nabla'\cdot\vec{P}}{\imath}\, d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \oint \frac{\sigma_b}{r}\, da' + \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_b}{r}\, d\tau'$$

$\boxed{\sigma_b = \vec{P}\cdot\hat{n}}$ → Surface bound charge density

$\boxed{\rho_b = -\nabla'\cdot\vec{P}}$ → volume bound charge density.

Gauss's Law in the presence of Dielectrics

Total charge density inside the dielectric.

$$\rho = \rho_b + \rho_f$$

ρ_f → free charge density
(It consists of electrons on a conductor or ions embedded in the dielectric material)

Acco. to Gauss's Law

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\varepsilon_0 \nabla\cdot\vec{E} = \rho = \rho_b + \rho_f$$

$$\nabla\cdot(\varepsilon_0\vec{E}) = -\nabla\cdot\vec{P} + \rho_f$$

$$\nabla\cdot(\varepsilon_0\vec{E}) + \nabla\cdot\vec{P} = \rho_f$$

$$\nabla\cdot(\varepsilon_0\vec{E} + \vec{P}) = \rho_f$$

$$\nabla \cdot (\vec{D}) = \rho_f$$

$$\boxed{\nabla \cdot \vec{D} = \rho_f}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

Electric displacement.

Differential from of Gauss Law in dielectrics.

$$\boxed{\oint \vec{D} \cdot \vec{da} = q_f}$$

Integral form of Gauss Law in dielectrics.

q_f = total free charge enclosed in the volume.

Linear Dielectrics

In linear dielectrics

polarisation $\propto$ total electric field

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

χ_e = electric susceptibility

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$= \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$$

$$= \varepsilon_0 (1 + \chi_e) \vec{E}$$

$$\boxed{\vec{D} = \varepsilon \vec{E}}$$

Scanned by CamScanner

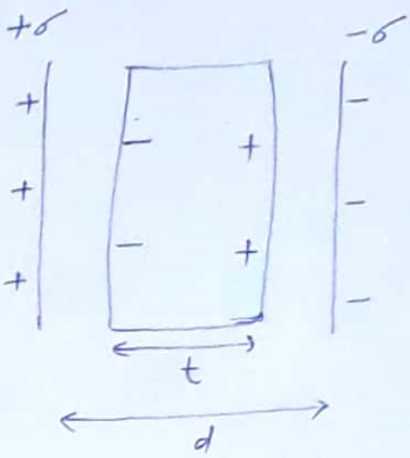
$$\varepsilon = \varepsilon_0(1+\chi_e)$$

$$\frac{\varepsilon}{\varepsilon_0} = 1+\chi_e$$

$$\boxed{\varepsilon_r = 1+\chi_e}$$

Relative permittivity OR Dielectric constant.

Capacitance of Parallel plate capacitor filled with dielectric



Electric filled b/w the plates outside the dielectric $= \vec{E_0}$

Electric filled inside the dielectric $= \vec{E}$

Net electric potential b/w the plates

$$V = Et + E_0(d-t)$$

$$= \frac{E_0}{K}t + E_0(d-t)$$

$$= E_0\left(d-t+t/K\right)$$

$$= \frac{q}{\varepsilon_0 A}\left(d-t+t/K\right)$$

$$E_0 = \frac{q}{\varepsilon_0 A}$$

$$q = CV$$

$$C = \frac{q}{V}$$

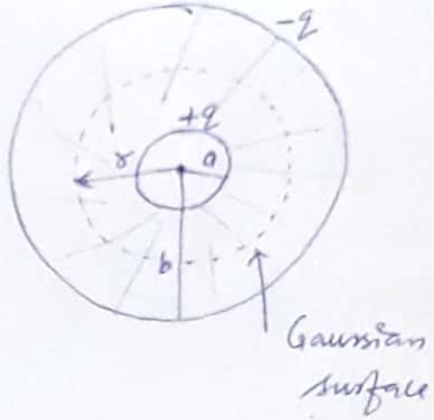
$$\boxed{C = \frac{\varepsilon_0 A}{d-t+t/K}}$$

If $t=d \Rightarrow C = \frac{\varepsilon_0 A}{d}K$

$$\boxed{C = C_0 K}$$

Capacitance of spherical capacitor filled with dielectric

Acc to Gauss's Law



$$\oint \vec{D}\cdot\vec{da} = q$$

$$\oint D_r\, da \cos 0 = q$$

$$D_r \oint da = q$$

$$D_r\, 4\pi r^2 = q$$

$$D_r = \frac{q}{4\pi r^2}$$

$$D = \varepsilon E$$

$$D_r = \varepsilon E_r$$

$$E_r = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$$

Potential difference b/w the two shells is

$$V = -\int_b^a E_r\, dr = -\int \frac{1}{4\pi\varepsilon}\frac{q}{r^2}\, dr$$

$$= -\frac{1}{4\pi\varepsilon}\, q \int \frac{1}{r^2}\, dr$$

$$= -\frac{1}{4\pi\varepsilon}\, q \left(\frac{-1}{r}\right)_b^a$$

$$= \frac{1}{4\pi\varepsilon}\, q \left(\frac{1}{r}\right)_b^a$$

$$= \frac{1}{4\pi\varepsilon}\, q \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{1}{4\pi\varepsilon}\, q \left(\frac{b-a}{ab}\right)$$

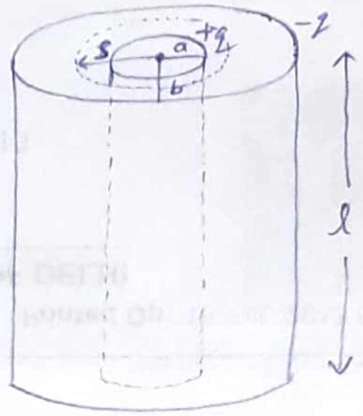
$$q = CV$$

$$C = \frac{q}{V}$$

$$\boxed{C = 4\pi\varepsilon\left(\frac{ab}{b-a}\right)}$$

Capacitance of Cylindrical capacitor filled with dielectric

Acco. to Gauss law –



$$\oint \vec{D}\cdot\vec{da} = q_{enclosed}$$

$$= \lambda l$$

$$\oint D_s\, da = \lambda l$$

$$D_s \oint da = \lambda l$$

$$D_s\, 2\pi s l = \lambda l$$

$$D_s = \frac{\lambda l}{2\pi s l} = \frac{\lambda}{2\pi s} \qquad E_s = \frac{1}{2\pi\varepsilon}\frac{\lambda}{s}$$

Potential difference b/w cylinders

$$V = -\int_b^a E_s\, ds = -\frac{1}{2\pi\varepsilon}\lambda\int_b^a \frac{1}{s}\, ds$$

$$= -\frac{1}{2\pi\varepsilon}\lambda\left(\ln s\right)_b^a = -\frac{1}{2\pi\varepsilon}\lambda\left(\ln a - \ln b\right)$$

$$= \frac{1}{2\pi\varepsilon}\lambda\left(\ln b - \ln a\right)$$

$$V = \frac{1}{2\pi\varepsilon}\lambda \ln\frac{b}{a}$$

$$q = CV$$

$$C = \frac{q}{V} = \frac{\lambda l}{V}$$

$$\boxed{C = \frac{2\pi\varepsilon l}{\ln(b/a)}}$$