Cosets

If $\exists$ G; and H is a proper subgroup of G such that $\|H\| < \|G\|$; Then let us define operation $g_i \in G$ and H as following

$$g_i \cdot H = \{ g_i \cdot h_j \ \forall \ h_j \in H \}$$

Case (I) If $g_i \in H \Rightarrow \quad g_i \cdot h_j \in H$

Thus by ~~rearra~~ rearrangement theorem $\quad g_i \cdot H = H$

Case (II) If $g_i \notin H \quad g_i \cdot H$ will have no element common to H.

Proof: If $g_i \notin H$ Let us assume $\exists$ at least one element which belongs to H which is contradiction to claim made in case II. Therefore for some h_j say

$\therefore \quad g_i h_j \in H$

$\because \quad h_j \in H \quad \therefore \quad h_j^{-1} \in H$

$\because \quad g_i h_j$ and $h_j^{-1} \in H \quad \therefore \quad g_i \cdot h_j \cdot h_j^{-1} \in H$

$\because$ Group binary operation is associative $\therefore \quad g_i \in H$.

This is against the assumption. Therefore its a contradiction

$\therefore$ If $g_i \notin H$ then $\quad g_i H \cap H = \phi$ (empty set)

Self check excercise: Show that if $g_i \notin H$ and $g_j \notin H$ & $g_i \cdot H$ then $\quad g_i \cdot H \cap g_j H = \phi$

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The collection of all $g_i.H$ such that "G" group is exhausted is called as left coset.

Similarly the collection $H.g_i$ such that "G" group is exhausted is called as right coset.

Example 1 Consider $G = \{E, C_4, C_4^2, C_4^3\}$; find left coset and right coset of subgroup $H = \{E, C_4^2\}$

Solution

Left coset $\quad H = \{E, C_4^2\}$

Let us take an element which belongs to G but does not belong to H. say C_4^3

$$\therefore \quad C_4^3 . \{E, C_4^2\} = \{C_4^3.E, C_4^3.C_4^2\}$$
$$= \{C_4^3, C_4\}$$

We can see all elements of group are exhausted.

Hence left coset are $\{E, C_4^2\}\{C_4^3, C_4\}$

Right coset $\quad H = \{E, C_4^2\}$

Let us take element $\in G \notin H$ say C_4.

$$\{E, C_4^2\}. C_4 = \{C_4, C_4^3\}.$$

$\therefore$ Right coset are $\{E, C_4^2\}$; $\{C_4^3, C_4\}$

Important: Coset is just collection of elements; hence ordering dosen't matter.

Question 1: Find right and left coset for all subgroup of symmetry of square.

Question 2: Find right and left coset for all subgroup of symmetry of triangle.

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