LECTURE 22: FM MODULATION AND THE PHASE LOCKED LOOP

In Lecture 4 we mentioned that a variation of ω in:

$$v = A\cos(\omega t = \phi) \tag{1}$$

resulted in frequency modulation of v. We now study frequency modulation (FM) in terms of the Fourier components in the frequency domain much as we decomposed an AM signal in Lecture 4.

As before, we use ω_c to denote the carrier angular frequency and ω_m to indicate the frequency of the "information" being encoded on the carrier. Again, only a single ω_m is considered for simplicity.

Let:

$$\omega = \omega_c(1 + k_f \cos\omega_m t) \tag{2}$$

We might be tempted to simply put this ω into the v expression:

$$v = A\cos[\omega_c(1 + k_f\cos\omega_m t)t + \phi] \tag{3}$$

Such a substitution would say that the angle of the outer cos at a given instant is the phase rate of change *at that instant* times the interval of time from t = 0 up to that instant. Actually, the cos accumulates angle (phase) as a sum of instantaneous rates times intervals of time. In other words:

$$\begin{aligned} v &= A\cos[\textstyle\int \omega(t)\,dt + \phi] \\ &= A\cos[\textstyle\int\omega_c(1 + k_f\cos\omega_m t)\,dt + \phi] \\ &= A\cos\left[\omega_c t + \frac{\omega_c k_f}{\omega_c}\sin\omega_m t + \theta_o + \phi\right] \end{aligned} \tag{4}$$

θ_o is the integration constant

$\omega_c k_f$ = frequency deviation

$$m_f = \frac{\omega_c k_f}{\omega_m} = \text{modulation index or deviation ratio}$$

For simplicity let $\theta_o + \phi = 0$.

$$v = A\cos\left[\omega_c t + \frac{\omega_c k_f}{\omega_m}\sin\omega_m t\right]$$

$$= A[\cos\omega_c t\cos(m_f\sin\omega_m t) - \sin\omega_c t\sin(m_f\sin\omega_m t)] \tag{5}$$

The cos of a sin and the sin of a sin can be reduced to a series of simple sin and cos terms.

$$v = A\cos\omega_c t\,\{\,[\,J_0(m_f) + 2J_2(m_f)\cos 2\omega_m t + 2J_4(m_f)\cos 4\omega_m t + \ldots\,]$$

$$-\sin\omega_c t\,[\,2J_1(m_f)\sin\omega_m t + 2J_3(m_f)\sin 3\omega_m t + \ldots\,]\,\} \qquad (6)$$

with J_n = bessel function of the first kind and order n.

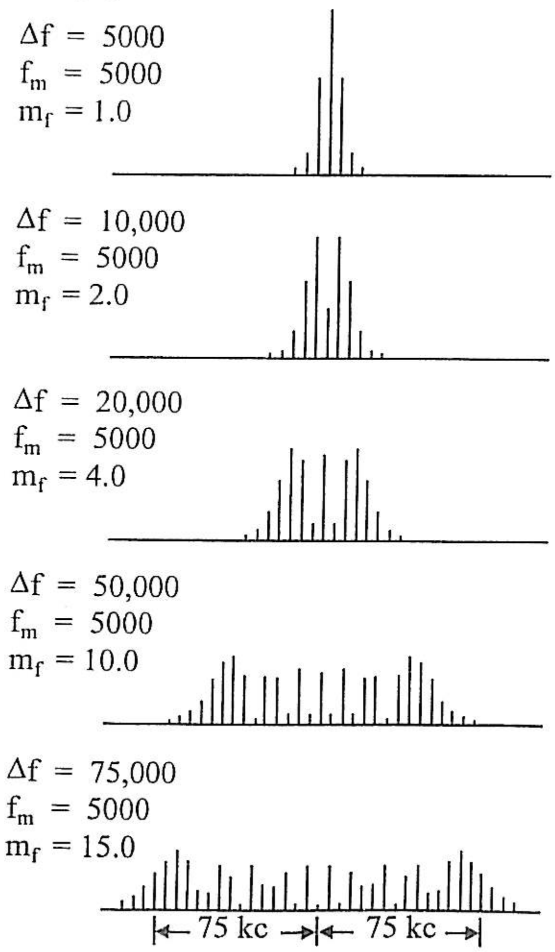
The product of two trig function terms reduces as usual to trig functions of the sum and difference frequencies. The resulting Fourier spectrum has all frequencies of the form:

$$\omega_c \pm n\omega_m$$

where n is any integer. The strength of each component depends on $J_n(m_f)$. Here we show some examples with:

$$\omega_m = 2\pi \times 5000$$

Δf is $\omega_c k_f/2\pi$. The vertical axis measures the Fourier component amplitude. Clearly the bandwidth required to pass an FM signal depends strongly on the modulation index.



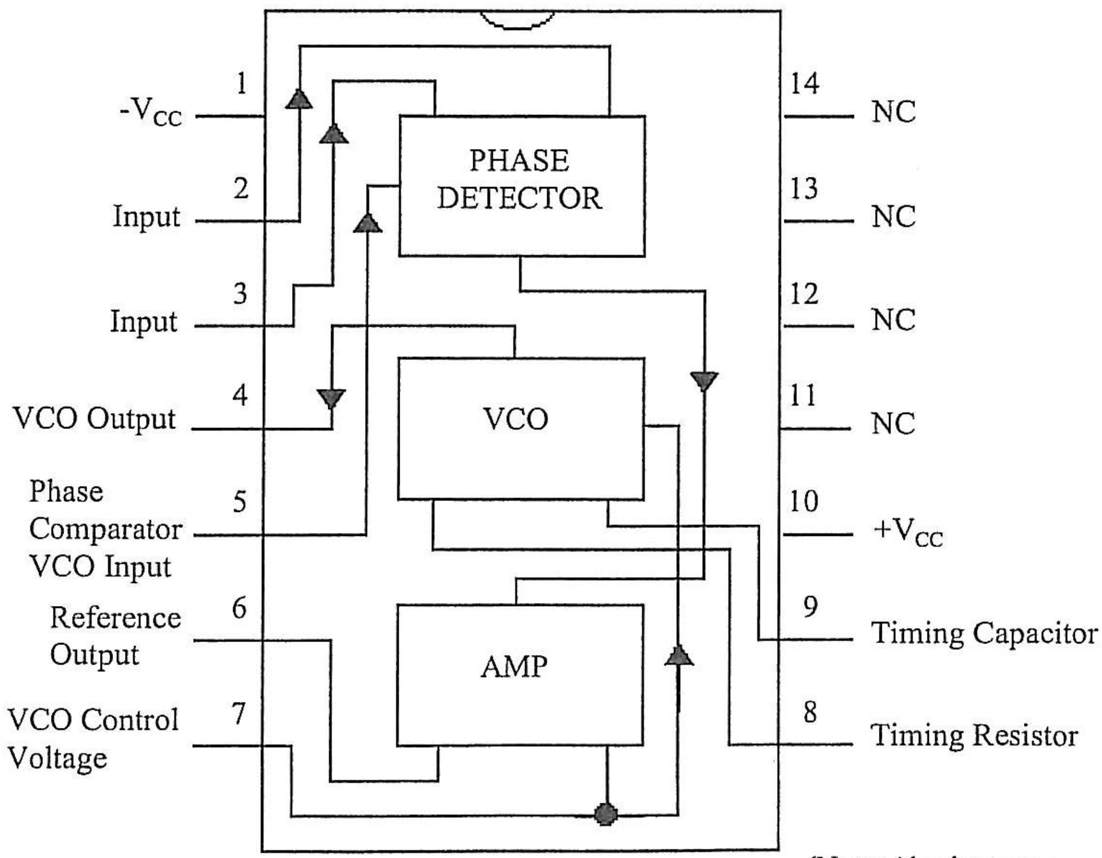
Frequency spectra of frequency-modulated waves, showing effect of varying the frequency deviation, for constant modulating frequency.

FM Demodulation: The Phase Locked Loop

There are several techniques for recovering the encoded information from an FM signal. We will use a "Phase Locked Loop" for this purpose. The phase locked loop is a versatile circuit with applicability far beyond FM demodulation.

The information given about the phase locked loop is borrowed from manufacturers applications notes. The selection process is aimed at collecting the minimum amount to allow basic understanding. The reading is not very easy since it is hard to understand the principles because of the jargon and the jargon definitions don't make too much sense without an understanding of the principles. Successive approximation techniques of learning will probably be needed.

There are several chips available with various extra features. We will use the simple and basic 565.



Top View

(Note: Absolute max supply voltage is ±12 volts.)

Sources:
(1) Linear Integrated Circuits, National
(2) Signetics Linear Applications

565 CONNECTIONS

Terminal 1 is $-V_{cc}$. The device can be operated either from symmetrical supplies such as ±12 volts or from a single supply such as +15 volts. In the latter case terminal 1 is connected to ground.

Terminals 2 and 3 form a differential input pair. The device does not have the gain of an op amp so $V_2 - V_3$ need not be almost zero to avoid saturation, i.e., no virtual equality. For single supply operation, terminals 2 and 3 should be biased to about 1/2 of $+V_{cc}$. For symmetric supplies, terminals 2 and 3 should be biased near ground. Do not exceed ±1 volts differential input! 10 mV for limiting; 5 KΩ input impedance.

Terminal 4 is the square wave output of the VCO. The output is between about the midpoint between $+V_{cc}$ and $-V_{cc}$ and $(+V_{cc} - 0.7)$ volts (i.e., like the 566 output.

Terminal 5 is the reference signal input to the phase comparator. This terminal is normally connected to terminal 4.

Terminal 6 is a dc reference about the same potential as terminal 7. Terminal 6 can be used with terminal 7 for driving differential inputs.

Terminal 7 is the amplified output of the phase detector. This terminal is also internally connected to the VCO. If desired, as in some synthesizer applications, this internal connection can effectively be broken (see Signetic Linear Applications 6-34). The dc component at pin 7 is about 4.5 volts when used with ± 6 volt supplies, and about 9 volts for ± 12 volt supplies.

Terminal 8 is connected through the VCO timing resistor to $+V_{cc}$. Choose $2\ K\Omega \leq R \leq 20\ K\Omega$. A 0.001 µf capacitor is normally connected from 8 to 7 for stability.

Terminal 9 is connected through the VCO timing capacitor to ac ground, usually either to chassis ground or $-V_{cc}$. A triangular wave voltage at the VCO frequency appears at this terminal.

Terminal 10 is $+V_{cc}$. Note: Do not exceed 24 volts for $+V_{cc} - (-V_{cc})$.

Components of the Phase Locked Loop

PHASE DETECTOR

A circuit which compares the input and reference signals and produces a voltage, called the error voltage, V_d, which is dependent on their relative phase difference. This circuit is also referred to as a phase comparator, a multiplier or a mixer. In the data sheets V_d is expressed as:

$$V_d = K_d \Delta\phi \text{ (really } V_d = K_c \cos(\phi_1 - \phi_2) \text{ as we see below)} \qquad (1)$$

where K_d is the phase detector gain factor, expressed in volts/radian, and $\Delta\phi$ is a phase difference in radians. Above a certain minimum input voltage, K_d is input-amplitude independent (see comment on limiting given under terminals 2 and 3 of the 565 connections).

Just to get the idea, consider how a multiplier functions as a phase detector. When $\sin \omega t$ and $\sin(\omega t + \phi)$ are multiplied:

$$\sin \omega t\,[\sin(\omega t + \phi)]$$

$$= \sin \omega t\,[\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$= \sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi$$

$$= \tfrac{1}{2}(1 - \cos 2\omega t) \cos \phi + \tfrac{1}{2} \sin 2\omega t \sin \phi$$

$$= \tfrac{1}{2} \cos \phi - \tfrac{1}{2} \cos 2\omega t \cos \phi + \tfrac{1}{2} \sin 2\omega t \sin \phi \qquad (2)$$

error-term

The double frequency terms are undesirable by-products which will be filtered out. This filter will be considered in the next lecture. Notice the difference in definition between ϕ and $\Delta\phi$ for this detector.

ϕ	$\tfrac{1}{2}\cos\phi$ = error voltage $\rightarrow$	$\Delta\phi = (K_d/V_d)^{-1}$
$0 < \phi < \pi/2$	+	+
$\phi = \pi/2$	0	0
$\pi/2 < \phi < \pi$	-	-

This table leads to $\Delta\phi = \pi/2 - \phi$. That is, there is zero error voltage if the input and reference signal are $\pi/2$ out of phase. The 565 phase detector behaves like this. The amplifier shown on page 4 is included in the K_d coefficient.

VOLTAGE CONTROLLED OSCILLATOR (VCO)

An oscillator whose frequency is determined, at least in part, by an applied control voltage. With zero applied voltage the oscillator operates at its free-running frequency or center frequency. For the 565 the resistor connected between pin 8 and $+V_{cc}$ and the capacitor connected between pin 9 and ground determine this center frequency through the equation:

$$f_o = \frac{1}{3.7 R_o C_o} \qquad (3)$$

or by the graph on the data sheets. The VCO conversion gain or VCO oscillator sensitivity, K_0 relates the frequency shift of the oscillator to the applied voltage. The units of K_0 are radians/sec/volt.

For the LM565 the product of K_0 and the K_d defined above is:

$$K_o K_d = \frac{33.6 f_o}{(+V_{cc}) - (-V_{cc})} \tag{4}$$

K_0K_d is called the loop gain. The loop gain may be reduced from the above value by connecting a resistor between pin 6 and pin 7.

Operation of the Phase Locked Loop

The phase locked loop compares an input signal to the VCO signal by means of the phase detector. The error signal from the phase detector is applied to the VCO in such a sense as to cause the VCO to have the same frequency as the input. Note that the VCO can follow the input frequency *exactly* since the error voltage is generated by a phase difference, not a frequency difference.

There are many uses of this chip. One application we can immediately understand is FM demodulation. The discussion is easiest in the time domain rather than the frequency domain. As the VCO tracks an FM signal back and forth, the phase error voltage mirrors the frequency excursions since it is the error voltage which drives the VCO to make these excursions. The error voltage amplitude oscillations then have the same period as the signal which modulates the FM carrier. The amplitude of the modulation determines the frequency deviation and thus the error voltage amplitude. Thus we see that the error voltage and the modulating signal have the same amplitude and frequency behavior, i.e., the phase locked loop has recovered the modulating signal from the FM signal.

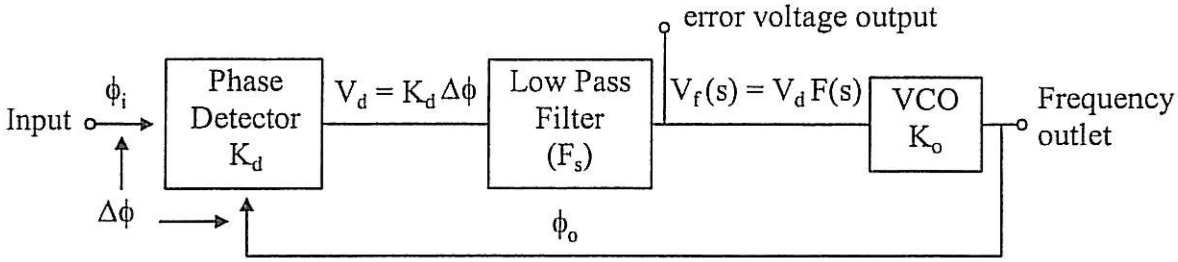
Now with some idea how a phase locked loop works, we can define a few more terms. The lock range, $2\omega L$ is the range of input frequencies about $2\pi f_0$ for which the VCO will follow the input frequency once the VCO is locked to the input. This term is also called the tracking range or the hold-in range. Since the biggest correction voltage occurs for $\Delta\phi = \pi/2$ we get:

$$\omega_L = K_o K_d \frac{\pi}{2}$$

If the loop is initially unlocked and some signal is applied to the input, that input signal must be within $\pm\omega_c$, the capture range, for the VCO to lock to the input. Clearly, $\omega_c \le \omega_L$, and in fact $\omega_c < \omega_L$. ω_c will be discussed more in the next lecture.

Linear Circuit Theory of the Phase Locked Loop

The low pass filter is that which was mentioned in the discussion of the phase detector. The output of the VCO is $\omega_o + \Delta\omega_o$, where ω_o is the free running frequency and $\Delta\omega_o$ is $K_o V_f$. Since frequency is the time derivative of phase, we can define a reference phase, ϕ_r for the whole derivation which constantly, and uninterestingly increases in time.



$$\frac{d\phi_r}{dt} = \omega_o \qquad (5)$$

we define ϕ_i and ϕ_o relative to ϕ_r:

$$\frac{d(\phi_r + \phi_i)}{dt} = \omega_o + \Delta\omega_i = \omega_i$$

the actual input frequency, and:

$$\frac{d(\phi_r + c + \phi_o)}{dt} = \omega_o + \Delta\omega_o \qquad (6)$$

where the C is included in the definition of ϕ_o so that $\Delta\phi$ can be defined as $\Delta\phi = \phi_i - \phi_o$ (recall that the phase detector puts out a nonzero result if the VCO and input are in phase).

From (6) and (5) we get:

$$\frac{d\phi_o}{dt} = \Delta\omega_o \qquad (7)$$

In this example of linear circuit theory, our variable is phase, not voltage. We proceed as usual to get function of "s" by Laplace Transforms. Since:

$$L\left(\frac{d\phi_o}{dt}\right) = sL\,(\phi_o)$$

we have:

$$s\phi_o(s) = \Delta\omega_o(s) = K_o V_f(s) = K_o K_d F(s) \Delta\phi(s) \qquad (8)$$

and:

$$\Delta\phi(s) = \phi_i(s) - \phi_o(s) \qquad (9)$$

so:

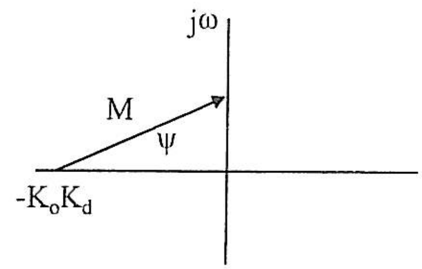
$$s\phi_o(s) = K_o K_d F(s)\,[\phi_i(s) - \phi_o(s)] \qquad (10)$$

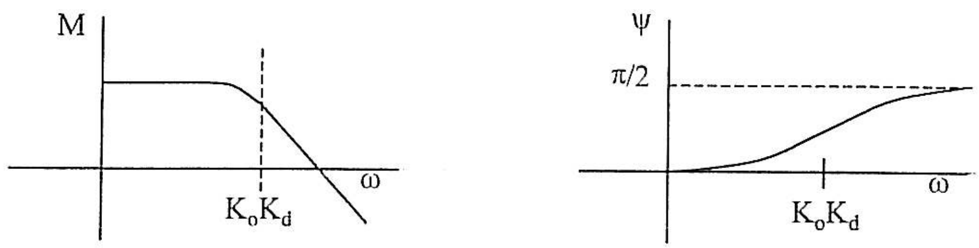
Finally, we obtain a system transfer function:

$$T(s) = \frac{\phi_o(s)}{\phi_i(s)} = \frac{K_o K_d F(s)}{s + K_o K_d F(s)} \qquad (11)$$

The simplest system has no filter, that is, $F(s) = 1$.

$$T(s) = \frac{K_o K_d}{s + K_o K_d}$$





The phase of T(s) is minus ψ since we are dealing with a pole. Remember that the Bode plot of M is a log-log plot. M, of course, does not go negative as a casual phase glance at the graph might suggest.