B.Sc. (H.) Physics (Section –II)
Semester IV

Elements of Modern Physics (2019-20)

Radioactivity

by
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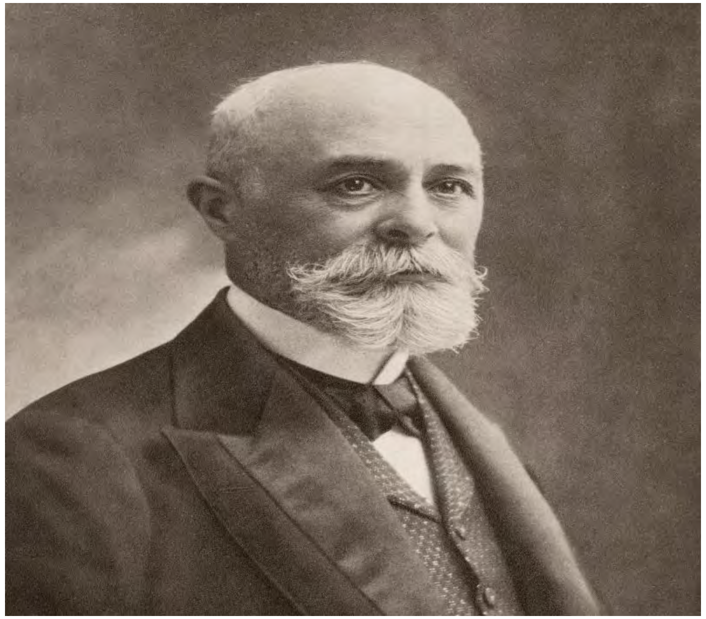
Radioactivity: Discovery



1896: Antoine Henri Becquerel French Engineer, Physicist, Nobel laureate was the first person to discover evidence of radioactivity.

1899: Ernest Rutherford, British physicist identified two kinds of rays emanating from radioactive substances and named them alpha and beta rays.

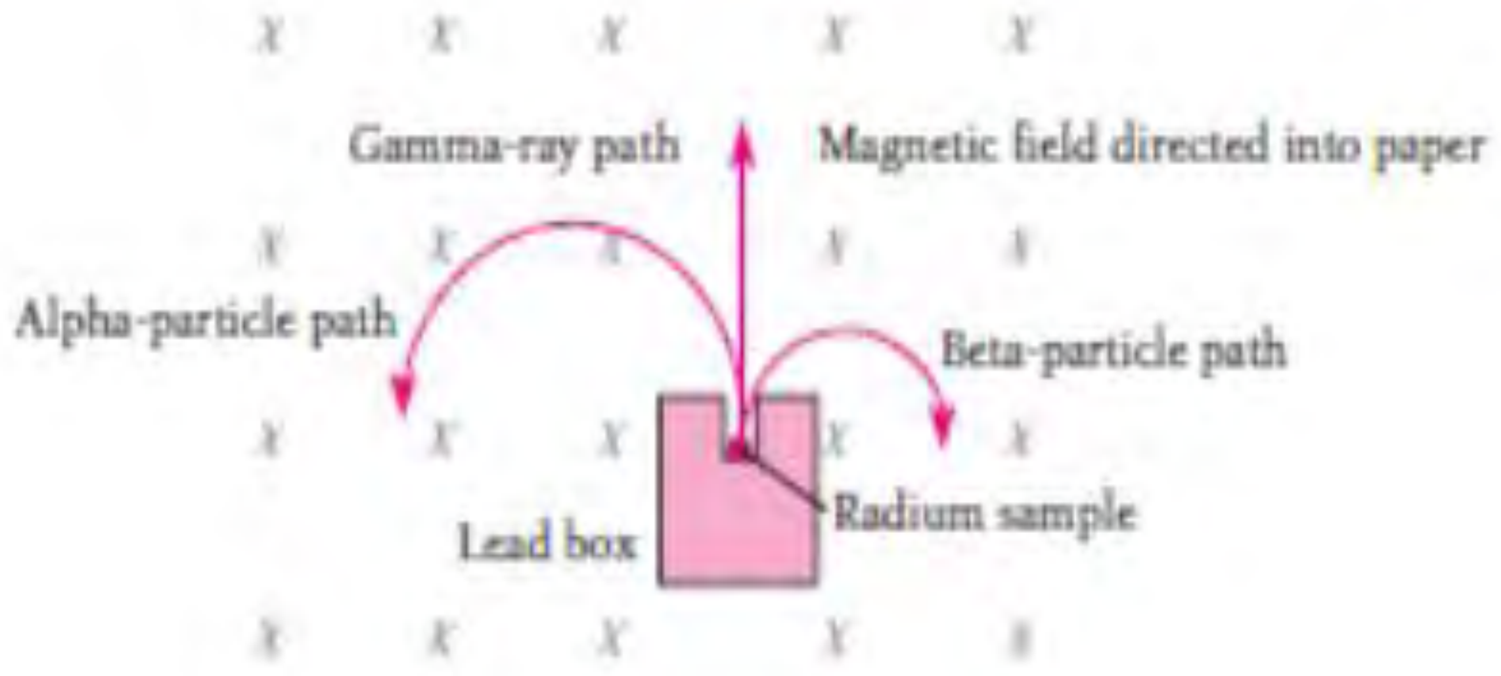
Photos: https://en.wikipedia.org/

Radioactivity: The phenomenon of spontaneous decay of a nucleus accompanied by the emission of alpha particles, beta particles or gamma-rays is known as radioactivity.
It can be either natural or artificial.
Most elements in nature have no radioactive isotopes.
Some elements have some stable isotopes and some radioactive ones, e.g., potassium.
Some elements have only radioactive isotopes, e.g., uranium.

Features extraordinary from the perspectives of classical physics:

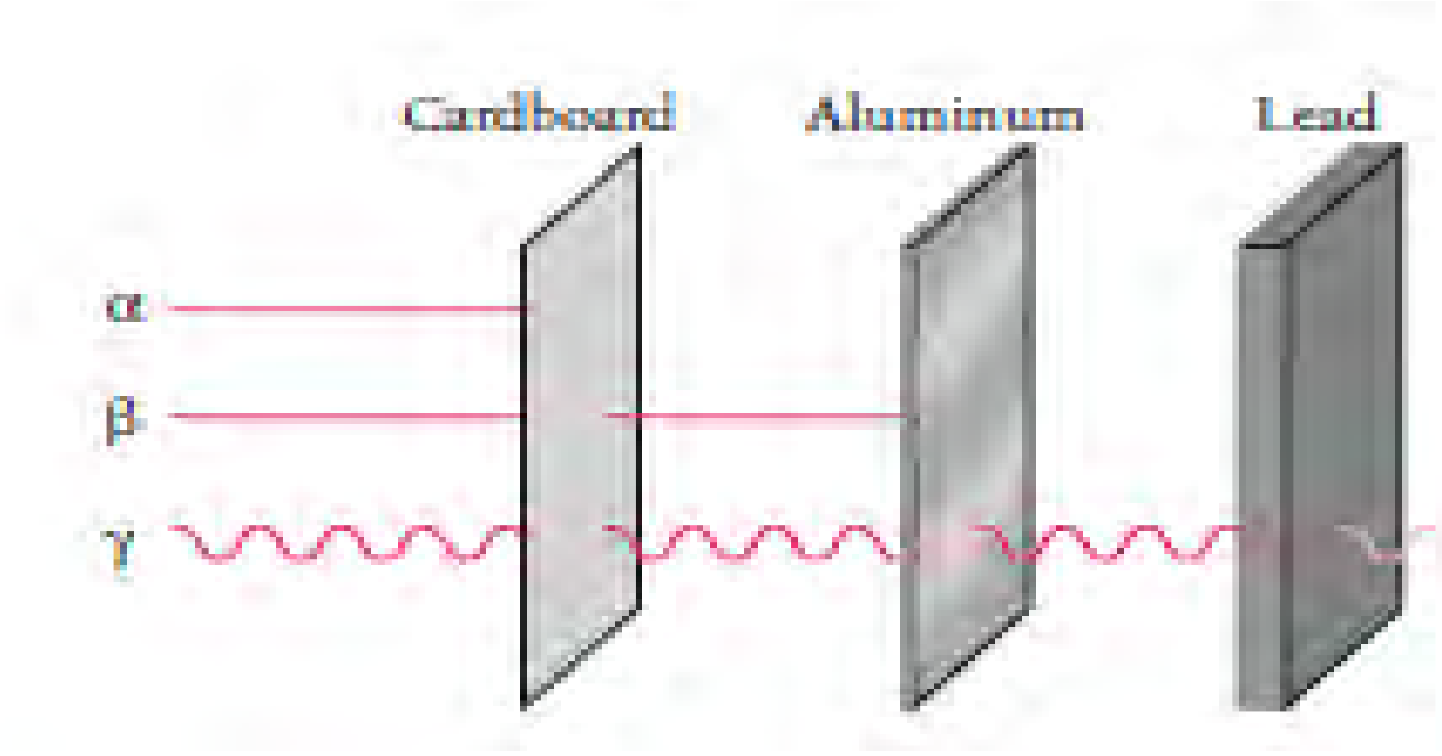
- When a nucleus undergoes alpha or beta decay, its atomic number Z changes and it becomes the nucleus of a different element. Thus the elements are not immutable.
- The energy liberated during radioactive decay comes from *within* individual nuclei without external excitation. Einstein's proposed equivalence of mass and energy could solve this puzzle.
- Radioactive decay is a statistical process that obeys the laws of chance. No cause-effect relationship is involved in the decay of a particular nucleus.

The experiments of Rutherford and his co-workers, distinguished three components in the penetrating radiations emitted from radionuclides. These components were called alpha, beta and gamma rays.

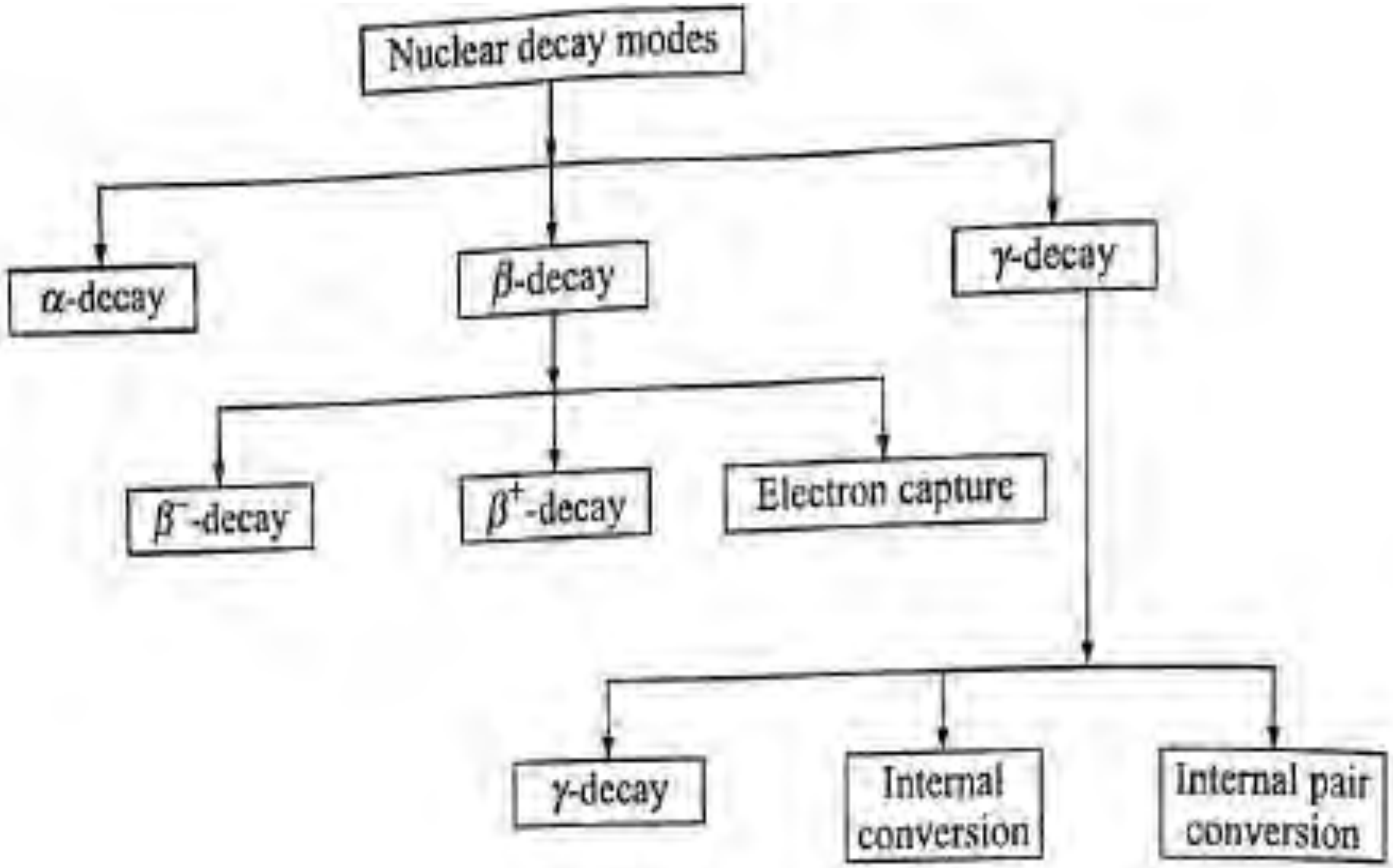


In the presence of a magnetic field:

- Alpha particles ($^4He_2$ nuclei) are deflected to the left, hence they are positively charged;
- Beta particles (electrons) are deflected to the right, hence they are negatively charged;
- Gamma rays (high-energy photons) are not affected, hence they are unchanged.



- Alpha particles from radioactive materials are stopped by a piece of cardboard.
- Beta particles penetrate the cardboard but are stopped by a sheet of aluminum.
- Even a thick slab of lead may not stop all the gamma rays.



Various decay modes of radioactive nuclei.

Table 12.1 Radioactive Decay[†]

Decay	Transformation	Example
Alpha decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	${}^{238}_{29}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$
Beta decay	${}^{A}_{Z}X \rightarrow {}_{Z+1}^{A}Y + e^{-}$	${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-}$
Positron emission	${}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}Y + e^{+}$	${}^{64}_{29}Cu \rightarrow {}^{64}_{28}Ni + e^{+}$
Electron capture	${}^{A}_{Z}X + e^{-} \rightarrow {}_{Z-1}^{A}Y$	${}^{64}_{29}Cu + e^{-} \rightarrow {}^{64}_{28}Ni$
Gamma decay	${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$	${}^{87}_{38}Sr^{*} \rightarrow {}^{87}_{38}Sr + \gamma$

[†]The * denotes an excited nuclear state and γ denotes a gamma-ray photon.

(i) **Gamma decay:** Alpha and beta decays of a radioactive nucleus usually leave the daughter nucleus in an excited state. If the excitation energy available with the daughter nucleus is not sufficient for further particle emission, it loses its energy by emitting electromagnetic radiations, also known as γ-rays. Mass and charge of the daughter nucleus remains the same as before the emission of γ-rays. One such example is

$$^{137}_{56}\mathrm{Ba}^{*} \rightarrow {}^{137}_{56}\mathrm{Ba} + \gamma$$

The star (*) on ^{137}Ba indicates that it is in excited state.

(ii) **Internal conversion:** In internal conversion process, an excited nucleus instead of emitting γ-rays directly transfers its excitation energy to one of the orbital electrons and the orbital electron is ejected as conversion electron. Mass and charge of the daughter nucleus remains the same as before the emission of the orbital electron. This is so because electron is ejected from the electronic orbits.

(iii) **Internal pair conversion:** If the excitation energy of the nucleus > 1.022 MeV, the nucleus can de-excite by emitting directly an electron and positron pair in its own Coulomb field. This process is known as *internal pair conversion.*

Activity: The rate at which the nuclei of the radioactive sample decay.

If *N* is the number of nuclei at a certain time t,

$$\textbf{Activity (R)} = -\, dN/dt$$

The minus sign is used to make *R* a positive quantity.

Units of activity:

- becquerel (**SI unit** of activity is named after Becquerel.)

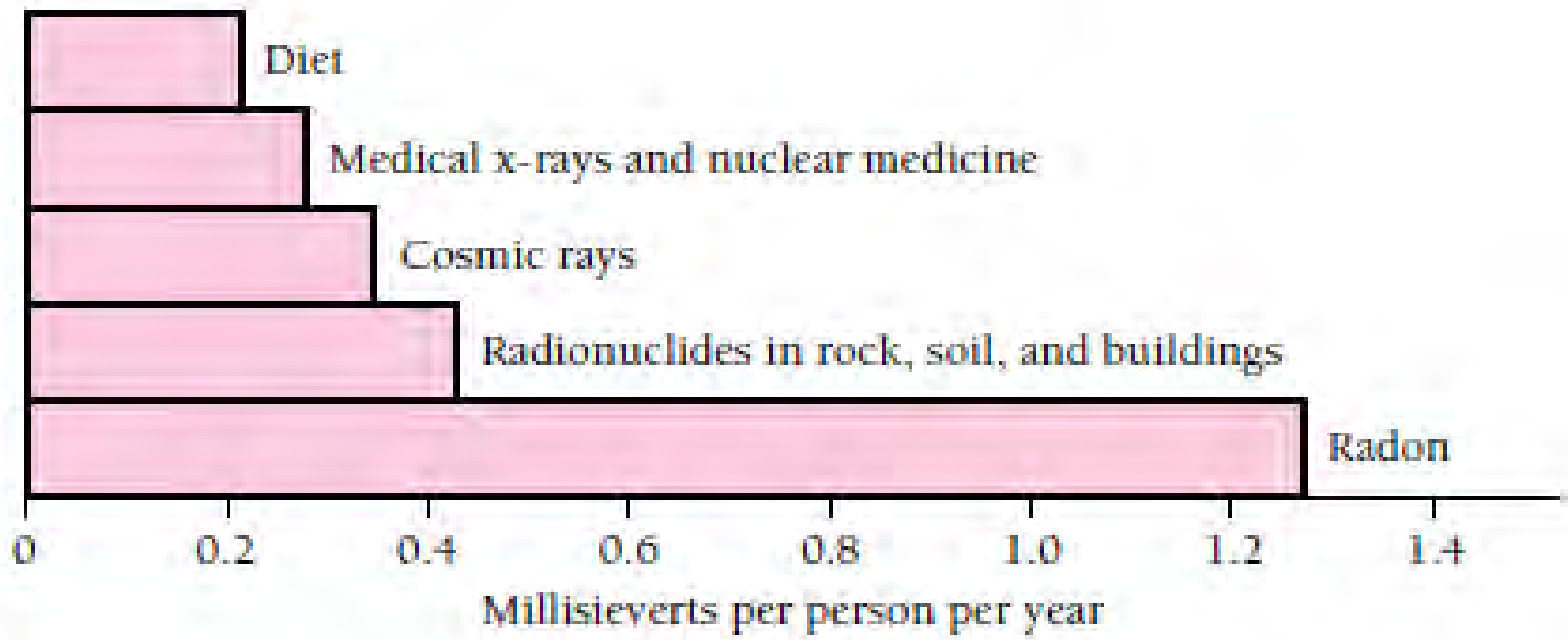
1 becquerel = 1 Bq = 1 decay/s

Megabecquerel (1 MBq =10^6 Bq), Gigabecquerel (1 GBq = 10^9 Bq)

- **curie** (Ci) (**Traditional unit** of activity.)

1 curie = 1 Ci = 3.70 * 10^10 decays/s = 37 GBq

Radiation dosage is measured in **sieverts** (Sv).
1 Sv is the amount of any radiation that has the same biological effect as those produced when 1 kg of body tissue absorbs 1 joule of x-rays or gamma rays.



The chief sources of radiation dosage averaged around the world.

Laws of radioactive disintegration:

- There is an equal probability for all nuclei of a radioactive element to decay.
- The rate of spontaneous disintegration of a radioactive element is proportional to the number of nuclei present at that time.

$$\frac{dN}{dt} \propto N \qquad (3.1)$$

N: number of atoms present at time t.

Removing proportionality sign, we get

$$\frac{dN}{dt} = -\lambda N \qquad (3.2)$$

λ: decay constant of the element.

Negative sign indicates that as t increases N decreases.

Rewriting Eq. (3.2) as

$$\frac{dN}{dt} = -\lambda dt \qquad (3.2)$$

Integrating both sides, we have $\int \frac{dN}{N} = -\lambda \int dt$

$$\ln(N) = -\lambda t + C \tag{3.4}$$

where C is constant of integration and is evaluated by the fact that at $t = 0$, number of atoms of the radioactive element is N_0. Using this condition, we get

$$C = \ln(N_0) \tag{3.5}$$

Substituting this value of C in Eq. (3.4), we get

$$\ln(N) = -\lambda t + \ln(N_0)$$

or

$$\ln \frac{N}{N_0} = -\lambda t$$

Thus,

$$\boxed{N = N_0 e^{-\lambda t}} \tag{3.6}$$

The exponential nature of this equation shows that it takes an infinite time for the whole of the radioactive material to disintegrate.

$$\mathbf{R} = -\,dN/dt = \lambda N$$

Substituting for N from Eq. (3.6)

$$N = N_0\, e^{-\lambda t}$$

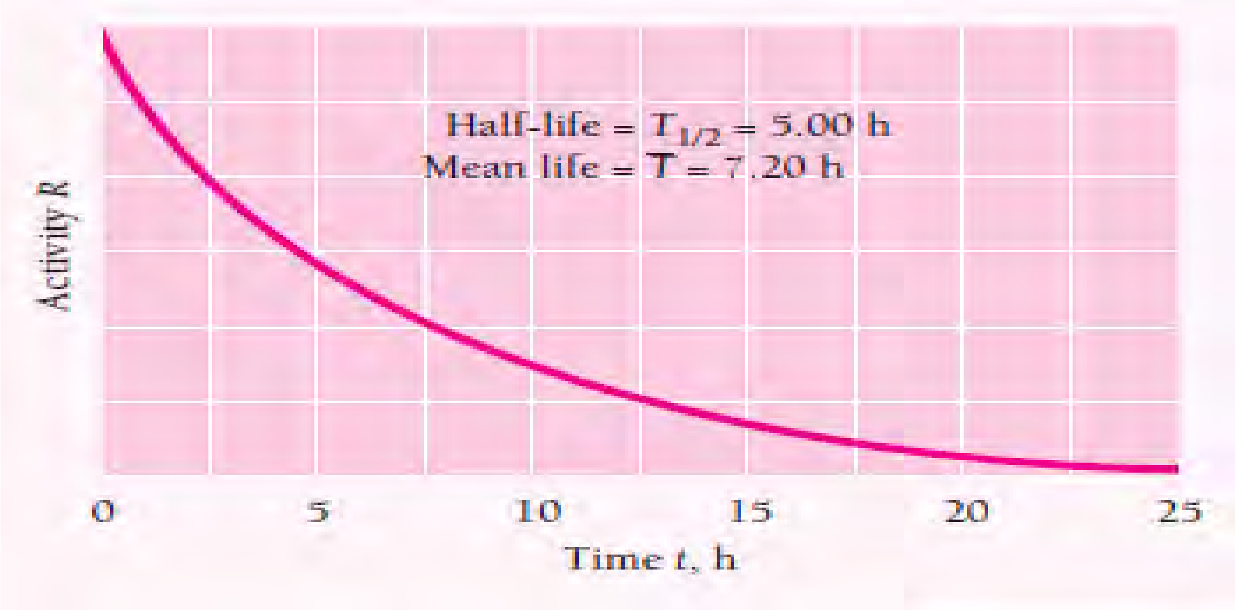
$$R = \lambda\, N_0\, e^{-\lambda t}$$

Substituting $R_0 = \lambda\, N_0$

$$R = R_0\, e^{-\lambda t}$$ ***Activity Law***

R_0 is the activity at $t = 0$.

The exponential factor shows that the activity is decreasing in the same fashion as N.



HALF-LIFE

Every radionuclide has a characteristic half-life.

After a half-life has elapsed, that is, when $t = T_{1/2}$, the activity R drops to ½ R_0 by definition.

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda T_{1/2}}$$

$$e^{\lambda T_{1/2}} = 2$$

Taking natural logarithms of both sides of this equation,

$$\lambda T_{1/2} = \ln 2$$

Half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The larger the decay constant, the greater the chance a given nucleus will decay in a certain period of time.

Average (Mean) Life

The average life is calculated by summing the lives of all the nuclei and dividing by the total number of nuclei. Suppose dN_1 nuclei decay in time t_1, dN_2 nuclei decay in time t_2, dN_3 nuclei decay in time t_3 and so on, then the average life or mean life will be

$$\tau = \frac{t_1 dN_1 + t_2 dN_2 + t_3 dN_3 + \cdots}{dN_1 + dN_2 + dN_3 + \cdots}$$

In integral form

$$\tau = \frac{\int_0^{N_0} t\, dN}{\int_0^{N_0} dN} = \frac{1}{N_0}\int_0^{N_0} t\, dN$$

$$dN = -\lambda N_0 e^{-\lambda t} dt$$

Now for $t = \infty$, $N = 0$ and for $t = 0$, $N = N_0$

Therefore,

$$\tau = -\frac{1}{N_0} \int_{\infty}^{0} \lambda t N_0 e^{-\lambda t} dt$$

$$\tau = \lambda \int_{0}^{\infty} t e^{-\lambda t} dt \tag{3.9}$$

The integral in Eq. (3.9) can be evaluated by parts

$$\int_0^{\infty} t e^{-\lambda t} dt = \left[t \frac{e^{-\lambda t}}{-\lambda} \right]_0^{\infty} - \int_0^{\infty} 1 \frac{e^{-\lambda t}}{-\lambda} dt$$

$$= (0 - 0) + \left[\frac{1}{\lambda} \times \frac{e^{-\lambda t}}{-\lambda} \right]_0^{\infty}$$

$$= -\frac{1}{\lambda^2}(0 - 1) = \frac{1}{\lambda^2}$$

Therefore, the value of average life from Eq. (3.9) is

$$\boxed{\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{0.693}} \tag{3.10}$$

Q. How long does it take for 60.0 percent of a sample of radon to decay? ($T_{1/2}$ = 3.8d.)

$$\frac{N}{N_0} = e^{-\lambda t} \qquad -\lambda t = \ln\frac{N}{N_0} \qquad \lambda t = \ln\frac{N_0}{N}$$

$$t = \frac{1}{\lambda}\ln\frac{N_0}{N}$$

Here $\lambda = 0.693/T_{1/2} = 0.693/3.82$ d and $N = (1 - 0.600)\,N_0 = 0.400N_0$, so that

$$t = \frac{3.82\text{ d}}{0.693}\ln\frac{1}{0.400} = 5.05\text{ d}$$

Q. Find the activity of 1.00 mg of radon, 222Rn, whose atomic mass is 222 u and $T_{1/2}$ = 3.8d.

Solution

The decay constant of radon is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(3.8\ \text{d})(86{,}400\ \text{s/d})} = 2.11 \times 10^{-6}\ \text{s}^{-1}$$

The number N of atoms in 1.00 mg of ^{222}Rn is

$$N = \frac{1.00 \times 10^{-6}\ \text{kg}}{(222\ \text{u})(1.66 \times 10^{-27}\ \text{kg/u})} = 2.71 \times 10^{18}\ \text{atoms}$$

Hence

$$R = \lambda N = (2.11 \times 10^{-6}\ \text{s}^{-1})(2.71 \times 10^{18}\ \text{nuclei})$$
$$= 5.72 \times 10^{12}\ \text{decays/s} = 5.72\ \text{TBq} = 155\ \text{Ci}$$

A piece of wood from the ruins of an ancient dwelling was found to have a ^{14}C activity of 13 disintegrations per minute per gram of its carbon content. The ^{14}C activity of living wood is 16 disintegrations per minute per gram. How long ago did the tree die from which the wood sample came?

$$R = R_0 e^{-\lambda t}$$

To solve for the age t we proceed as follows:

$$e^{\lambda t} = \frac{R_0}{R} \qquad \lambda t = \ln \frac{R_0}{R} \qquad t = \frac{1}{\lambda} \ln \frac{R_0}{R}$$

the decay constant λ of radiocarbon is $\lambda = 0.693/T_{1/2} = 0.693/5760$ y.

$R_0/R = 16/13$ and so

$$t = \frac{1}{\lambda} \ln \frac{R_0}{R} = \frac{5760 \text{ y}}{0.693} \ln \frac{16}{13} = 1.7 \times 10^3 \text{ y}$$

Reference Books:

- Concepts of Modern Physics, Arthur Beiser, 2002, McGraw-Hill.
- Introduction to Nuclear and Particle Physics, V. K. Mittal, R. C. Verma, S. C. Gupta, 3rd Edition, PHI Learning Private Limited.

Thanks