Nuclear Models:

(i) Liq. drop model: Weizsacker (1935) proposed on the basis of experimental facts that a nucleus resembles a drop of liquid.

Similarities between Liq. drop and Nucleus:

(i) Nuclear forces are analogous to the surface tension of a liq.

(ii) The nucleons behave in a manner similar to that of molecules in a liq. drop.

(iii) The density of the nuclear matter is independent of A, showing resemblance to liq. drop where the density of a liq is independent of the size of the drop.

(iv) The constant BE/nucleon is analogous to the latent heat of vaporization.

(v) The disintegration of nuclei by the emission of particles is analogous to the evaporation of molecules from the surface of liq.

(vi) The absorption of bombarding particles by a nucleus corresponds to the condensation of drops.

(vii) The energy of the nuclei corresponds to internal thermal vibrations of drop molecules.

Assumptions:

(i) The nucleus consists of incompressible matter.

(ii) The nuclear force is identical for every nucleon.

(iii) The nuclear force saturates.

(iv) In an equilibrium state, the nuclei of atom remains spherically symmetric under the action of strong attractive nuclear forces.

(4) proposed

certain aspects of nuclear behavior. This analogy was proved by George Gamow in 1929 and developed in detail by CF von Weizsacker in 1935.

Let the energy associated with each nucleon-nucleon bond has some value U. [U is -ve since attractive forces are involved but BE is considered a +ve quantity for convenience.] Each bond energy is shared by two nucleons, each has binding energy $\frac{1}{2}U$. When an assembly of spheres of same size is packed together into the smallest volume, each interior sphere has 12 other spheres in contact with it. Hence each interior nucleon has a BE of $12(\frac{1}{2}U)$. If all A nucleons in a nucleus were in its interior, the total BE of the nucleus would be

$E_V = 6AU$

$a_1 = 6U$

$\Rightarrow E_V = a_1 A$ — (I) (Volume energy)

Actually some nucleons are on the surface of every nucleus and therefore have fewer than 12 neighbors. The number of such nucleons depends on the surface area of the nucleus, $4\pi R^2 = 4\pi R_0^2 A^{2/3}$ where R is the nuclear radius. Then

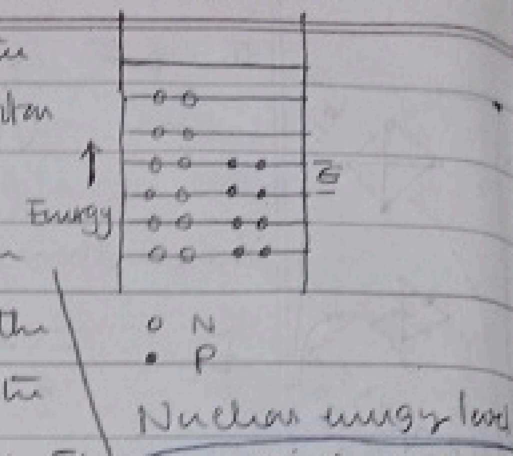
Surface energy, $E_S = -a_2 A^{2/3}$ — (II)

* most significant for the lighter nuclei since a greater fraction of their nucleons are on the surface.

→ The Coulomb term represents the energy incorporated in the nucleus as a result of the +ve charge present in the nucleus. The Coulomb energy of a nucleus is the W.D. that must be done to bring together Z protons from infinity into a spherical aggregate the size of the nucleus.

Corrections:

(1) The <u>asymmetry term</u> reflects the stability of nuclei with the proton and neutron nos being approx. equal and depends on the <u>neutron excess (N-Z)</u> in the nucleus. It decreases with the increasing nuclear BE. The reduction in BE for higher A nuclei is directly prop. to $(N-Z)^2$ and inversely proportional to A.



$$B_4 \propto \frac{(N-Z)^2}{A}$$

$$\Rightarrow B_4 = -a_4 \frac{(A-2Z)^2}{A} \quad -(\text{IV})$$

$$\boxed{\begin{aligned} & N-Z \\ &= A+N-Z-A \\ &= (A-Z-(A-N)) \\ &= A-Z-Z \\ &= (A-2Z) \end{aligned}}$$

(2) The last correction term arises from the tendency of <u>proton pairs and neutron pairs to occur</u>. Even-even nuclei are most stable and have higher BEs and odd-odd nuclei have both unpaired protons and neutrons have relatively low BEs. The pairing energy E_p is +ve for <u>even-even</u>, zero for odd-even and even-odd nuclei, and -ve for odd-odd nuclei, and seems to vary with A as $A^{-3/4}$.

$$E_p = (\pm, 0)\frac{a_5}{A^{+3/4}} \quad -(\text{V})$$

Thus the semi-empirical B-E formula first obtained by CF von Weizsacker in 1935,

$$\boxed{E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z(Z-1)}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} (\pm, 0)\frac{a_5}{A^{3/4}}}$$

It agrees better with observed binding energy.

* This formula reproduces masses of various nuclei quite accurately, but does not account for all the features of nuclear BE.

we can write semi empirical mass formula

$$M(Z,A) = Zm_p + Nm_n - E_b$$

$$= Zm_p + Nm_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z(Z-1)}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} (\mp, 0) \frac{a_5}{A^{3/4}}$$

$a_1 = a_v$

$a_2 = a_s$

$a_3 = a_c$

$a_4 = a_a$

Note: Isobars are nuclides that have same mass no. A.
The semi empirical formula can predict the atomic number Z_0 of most stable isobar for given mass no. A.
(* different proton)

$$\cancel{M(Z,A) = m_n(Z+N) - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A-2Z)^2}{A} (\mp, 0) \frac{a_5}{A^{3/4}}}$$

$$\cancel{= A(m_n - a_1) + a_2 A^{2/3}}$$

—o—

$$E_b/_A = a_1 - a_2 \frac{A^{2/3}}{A} - a_3 \frac{Z(Z-1)}{A\, A^{1/3}} - a_4 \frac{(A-2Z)^2}{A\, A} (\pm, 0) \frac{a_5}{A\, A^{3/4}}$$

$$\Rightarrow E_b/_A = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}} - a_4 \frac{(A-2Z)^2}{A^2} (\pm, 0) \frac{a_5}{A^{7/4}}$$

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nucleus/

* Binding Energy:

1. Mass defect: Mass of an atom is concentrated in the nucleus, which is at the center of the atom. It is observed that the mass of the nucleus is always less than the sum of the mass of all nucleons present in the nucleus. This difference in mass is known as the mass defect (Δm).

$$\Delta m = ZM_p + NM_n - \underline{M'(Z,N)}$$

mass of the bare nucleus

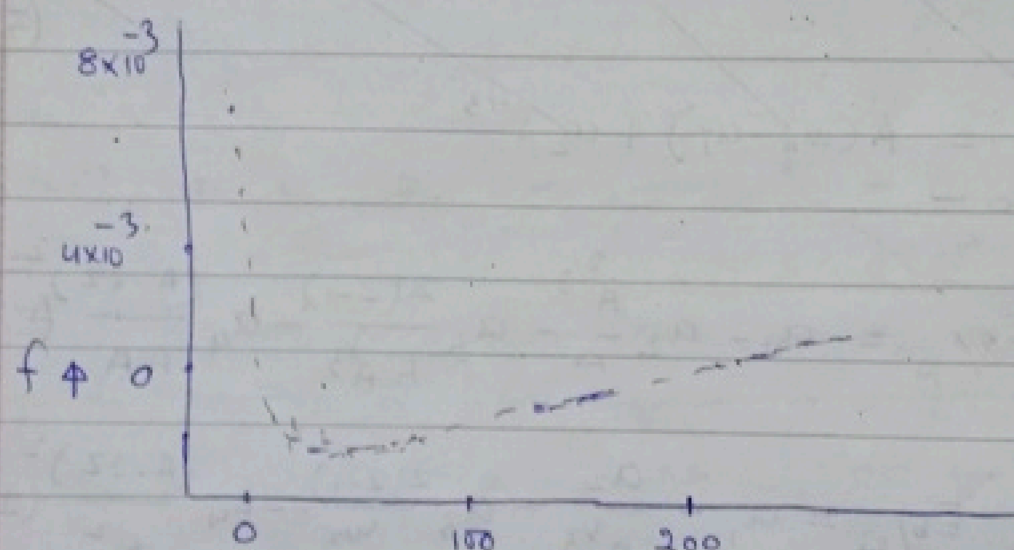
⑧

2. Packing fraction : Mass defect does not convey much information about nuclear stability. The term packing fraction gives better information about the nuclear stability.

$$\text{packing fraction, } f = \frac{\text{At. mass} - \text{Mass no.}}{\text{Mass no.}} \quad \underline{(\Delta m)}$$

$$\Rightarrow \boxed{f = \frac{\Delta m}{A}}$$

The smaller the ~~pak~~packing fraction the more stable is the nucleus and vice versa.



* loosely bound i. ✓ ~~A<16~~ A : decreasing f till A=16

ii. $16 < A < 180$, $f = -ve$; Strongly bound

ii. $60 < A < 80$, $f = -ve$; need supply external energy for breaking

iv. $A > 180$, $f = +ve$, $A > 235$; unstable

3. Binding energy : The mass of any permanently stable nucleus is found to be less than the sum of the mass of the neutrons and the protons which it contains. The fact is accounted for by the conversion of a part of the mass energy of the

(11)

(ii) Radioactive Nucleus; A molecule in the liq drop evaporates by gaining energy from its neighbouring molecules during the process of collision by the increase of the temperature and similarly a nucleon or a group of nucleons may leave the nucleus by gaining energy from the neighbouring nucleons during the process of collision, thus exhibiting the phenomenon of radioactivity

(iii) Nuclear fission —

failures:

(i) fail to explain high stability of nucleus with magic nos.

(ii) does not explain the measured spin and magnetic moments of the nuclei.

** Atomic no. of the most stable nucleus for a given Mass no. on liq drop Model:

for a stable nucleus

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + a_5 A^{-3/4}$$

$$\left(\frac{\partial E_b}{\partial Z}\right)_A = 0$$

$$\Rightarrow 0 = 0 + 0 - a_3 \frac{\cancel{2}Z}{A^{1/3}} - a_4 \frac{2(A-2Z)}{A}(-\cancel{2})$$

$$\Rightarrow -\frac{a_3}{A^{1/3}} Z + \frac{2a_4}{A}(A-2Z) = 0$$

$$\Rightarrow -\frac{a_3}{A^{1/3}}Z + \frac{2}{A}a_4(A-2Z) = 0$$

$$\Rightarrow +\frac{a_3}{A^{1/3}}Z + \frac{2}{A}a_4(2Z) = 2a_4$$

$$\Rightarrow Z\left(\frac{a_3}{A^{1/3}} + \frac{4a_4}{A}\right) = 2a_4$$

$$\Rightarrow Z = \frac{2a_4}{\frac{a_3}{A^{1/3}} + \frac{4a_4}{A}} = \frac{2a_4}{\frac{4a_4 + a_3A^{2/3}}{A}}$$

$$\frac{A^1}{A^{1/3}} \quad 1-\frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$$

$$\Rightarrow Z = \frac{A}{2 + \frac{a_3}{2a_4}A^{2/3}}$$

$a_3 = 0.7053\,MeV$

$a_4 = 23.702\,MeV$

0.01487849

$\equiv 0.015$

$$\boxed{Z = \frac{A}{2 + 0.015A^{2/3}}}$$

for light nuclei, having small A the term $0.015A^{2/3}$ can be neglected,

$$\boxed{Z = \frac{A}{2}}$$ experimentally confirmed.

- ${}^{6}_{2}He$, ${}^{6}_{4}Be$, ${}^{6}_{3}Li$ stable nucleus *

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Note: