BUSINESS MATHS

The subject of Differential Calculus deals with the problem of finding the instantaneous or local rate of change of a fun^n.

DIFFERENTIATION

1. $\frac{d}{dx}(x^n) = nx^{n-1}$ Eg ↓

2. Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

Eg - next page

Eg. 1. ↑ $x^{3/2} = \frac{3}{2}x^{3/2-1} = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$

$\frac{1}{\sqrt{x}} = \frac{d}{dx}(x)^{-1/2} = -\frac{1}{2}x^{-3/2}$

3. In case of a constant: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

Eg. $10x^2 = 10\frac{d}{dx}(x^2) = 10(2x) = 20x$

4. Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Eg If $y = \frac{3x+1}{x-1} = \frac{dy}{dx} = \frac{(x-1)\frac{d}{dx}(3x+1) - (3x+1)\frac{d}{dx}(x-1)}{(x-1)^2}$

$= \frac{(x-1)3 - (3x+1)(1)}{(x-1)^2} = -\frac{4}{(x-1)^2}$

5. Chain Rule: If $y = f(u)$ & $u = g(x)$ we can say

$y = f[g(x)]$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

February

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Eg. If $y = u^2$ & $u = 4x^3 + 7x + 1$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \frac{d}{du}(u^2) \cdot \frac{d}{dx}(4x^3 + 7x + 1)$$

$$= 2u(12x^2 + 7)$$

$$= 2(4x^3 + 7x + 1)(12x^2 + 7)$$

Eg. of Product Rule

If $y = (2x^2 + 3x + 1)(5x + 1)$

$$\frac{dy}{dx} = (4x + 3)(5)$$

$$= 20x + 15$$

✓ $2x^2 + 3x + 1 \frac{d}{dx}(5) + 5x + 1 \cdot (4x + 3)$

$= 10x^2 + 15x + 5 + 20x^2 + 4x + 15x + 3$

$= 30x^2 + 34x + 8$

Property

General :- $\frac{d}{dx}(e^x) = e^x$

$\log x = \frac{1}{x}$

Q. If $y = \frac{\log x}{x}$

$$\frac{dy}{dx} = \frac{x \frac{d}{dx}(\log x) - \log x \frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\left(\frac{1}{x}\right) - \log x \cdot 1}{x^2}$$

$$= \frac{1 - \log x}{x^2}$$

January

Mo	Tu	We	Th	Fr	Sa	Su
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24	25	26	27	28	29	30
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Q. If $y = e^{2x^2+3x+7}$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = e^{2x^2+3x+7}\frac{d}{dx}(2x^2+3x+7)$$

$$= e^{2x^2+3x+7}(4x+3)$$

Q. If $y = x^2\log(3x+7)$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = x^2\frac{d}{dx}[\log(3x+7) + \log(3x+7)\frac{d}{dx}(x^2)]$$

$$= x^2\left(\frac{1}{3x+7}\right)(3) + [\log(3x+7)](2x)$$

$$= \frac{3x^2}{3x+7} + 2x\log(3x+7)$$

$\boxed{a^x = a^x \log_e a}$

Q. If $y = x^3 + 4^x + \log x$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(4^x) + \frac{d}{dx}(\log x)$$

$$= 3x^2 + 4^x\log 4 + \frac{1}{x}$$

Eg of Implicit Differentiation

Q. Find dy/dx when $x^3+y^3 = xy$

x and y both are given

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = \frac{d}{dx}(xy)$$

$$3x^2 + 3y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$(3y^2 - x)\frac{dy}{dx} = y - 3x^2$$

$$\frac{dy}{dx} = \frac{y-3x^2}{3y^2-x}$$

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Logarithmic Differentiation

This method is used to differentiate functions of the form $y = f(x)^{g(x)}$. With this method, we first take the natural lograrithm of both sides.

Eg If $y = x^x$, find $\frac{dy}{dx}$.

$$\log y = \log(x^x) = x \log x$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \log x\,(1) = 1 + \log x$$

Multiplying both sides by y & substituting x^x for y

$$\therefore \frac{dy}{dx} = y[1+\log x] = x^x[1+\log x]$$