March 31, 2020

Assignment-I (Chapter 2 (Hogg et al.)) Dr. SUSHIL KUMAR AZAD

1. Let X and Y be r.v.'s with $\mu_1 = 1$, $\mu_2 = 4$, $\sigma_1^2 = 4$, $\sigma_2^2 = 6$, $\rho = \frac{1}{2}$. Find the mean and variance of the r.v. $Z = 3X - 2Y$.

2. Determine the correlation coefficient of the r.v.'s X and Y if $var(X) = 4$, $var(Y) = 2$, and $var(X+2Y) = 15$.

3. Find the variance of the sum of 10 r.v.'s if each has variance 5 and if each pair has correlation coefficient 0.5.

4. Let X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} 15x_1^2 x_2, & 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the marginal pdfs and compute $P(X_1 + X_2 \le \frac{1}{2})$.

5. Let (X_1, X_2) be the random vector defined on the unit square with uniform probability distribution, i.e. the pdf of X_1 and X_2 is

$$f(x_1, x_2) = \begin{cases} 1, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0. & \text{elsewhere} \end{cases}$$

Find the pdf of $Z = X_1 + X_2$.

6. Suppose X_1 and X_2 are r.v.'s of the discrete type that the joint pmf

$$p(x_1, x_2) = \begin{cases} \frac{(x_1 + 2x_2)}{18}, & (x_1, x_2) = (1,1), (1,2), (2,1), (2,2) \\ 0, & \text{elsewhere} \end{cases}$$

Determine the conditional mean and variance of X_2, given $X_1 = x_1$, for $x_1 = 1$ or 2. Also compute $E(3X_1 - 2X_2)$.

Continued

7. Let us choose at random a point from the interval $(0,1)$ and let the r.v. X_1 be equal to the number that corresponds to that point. Then choose a point at random from the interval $(0, x_1)$, where x_1 is the experimental value of X_1; and let the r.v. X_2 be equal to the number that corresponds to this point.

(a) Make assumptions about the marginal pdf $f_1(x_1)$ and the conditional pdf $f_{2|1}(x_2|x_1)$.

(b) Compute $P(X_1+X_2 \geq 1)$

(c) Find the conditional mean $E(X_1|x_2)$.

8. Let X and Y be r.v.'s with the space $S=\{(0,0), (1,1), (1,0), (1,-1)\}$. Assign positive probabilities to the four points of S so that the correlation coefficient is equal to zero. Are X and Y independent?

9. Let X and Y have the joint pmf

$$p(x,y)=\begin{cases} \frac{1}{7}, & \text{if } (x,y)=(0,0),(1,0),(0,1),(1,1), \\ & (2,1),(1,2),(2,2) \\ 0 & \text{elsewhere.} \end{cases}$$

Find the correlation coefficient, ρ_{XY}.

10. Let $\sigma_1^2=\sigma_2^2=\sigma^2$ be the common variance of X_1 and X_2, and let ρ be the correlation coefficient of X_1 and X_2. Show that for $k>0$,

$$P\left[|(X_1-\mu_1)+(X_2-\mu_2)| \geq k\sigma\right] \leq \frac{2(1+\rho)}{k^2}$$