

B.Sc. (H.) Physics (Section –II)  
Semester IV

Elements of Modern Physics (2019-20)

Alpha Decay

by  
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## Alpha decay

It occurs in heavy nuclei ( $A > 210$ ) as a means of increasing their stability by reducing their size.

Alpha particles are emitted rather than individual protons or  ${}^3\text{He}_2$  nuclei because of their high binding energies.

Energy  $Q$  released when various particles are emitted by a heavy nucleus is given by

$$\text{(Disintegration energy)} \quad Q = (m_i - m_f - m_x)c^2$$

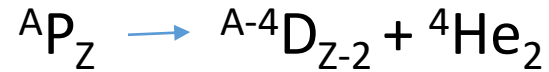
$m_i$  : mass of initial nucleus,  $m_f$  : mass of final nucleus,  $m_x$  : particle mass.

Alpha decay in  ${}^{232}\text{U}_{92}$  is accompanied by the release of 5.4 MeV,

Proton needs 6.1 MeV to be supplied from outside and

${}^3\text{He}_2$  nucleus needs 9.6 MeV to be to be supplied from outside.

## Energy of alpha particles emitted during alpha decay



From mass-energy conservation

$$m_p c^2 = m_D c^2 + m_\alpha c^2 + K_D + K_\alpha,$$

$m_p$  : mass of Parent atom,  $m_D$  : mass of Daughter atom

$K_D$  : Kinetic energy of Daughter atom,  $K_\alpha$  : Kinetic energy of  $\alpha$  particle.

Here, we have assumed that the parent atom is at rest.

$$\text{(Disintegration energy) } Q = K_D + K_\alpha = [m_p - (m_D + m_\alpha)] c^2$$

Conservation of linear momentum demands

$$m_\alpha v_\alpha = m_D v_D. \tag{1}$$

$v_D$  : speeds of daughter atom,  $v_\alpha$  speed of alpha particle.

## Energy of alpha particles emitted during alpha decay (contd.)

$$K_{\alpha} = m_{\alpha} v_{\alpha}^2 / 2, \quad K_D = m_D v_D^2 / 2$$

Substituting for  $v_D$  from Eq. (1)

$$K_D = m_D (m_{\alpha} v_{\alpha} / m_D)^2 / 2 = m_{\alpha} v_{\alpha}^2 (m_{\alpha} / m_D) / 2$$

$$m_D K_D = m_{\alpha} K_{\alpha}$$

$$Q = K_D + K_{\alpha} = K_{\alpha} (1 + m_{\alpha} / m_D)$$

Since,  $m_{\alpha} = 4$  and  $m_D = A-4$ ,

$$K_D = (4/(A-4)) K_{\alpha}$$

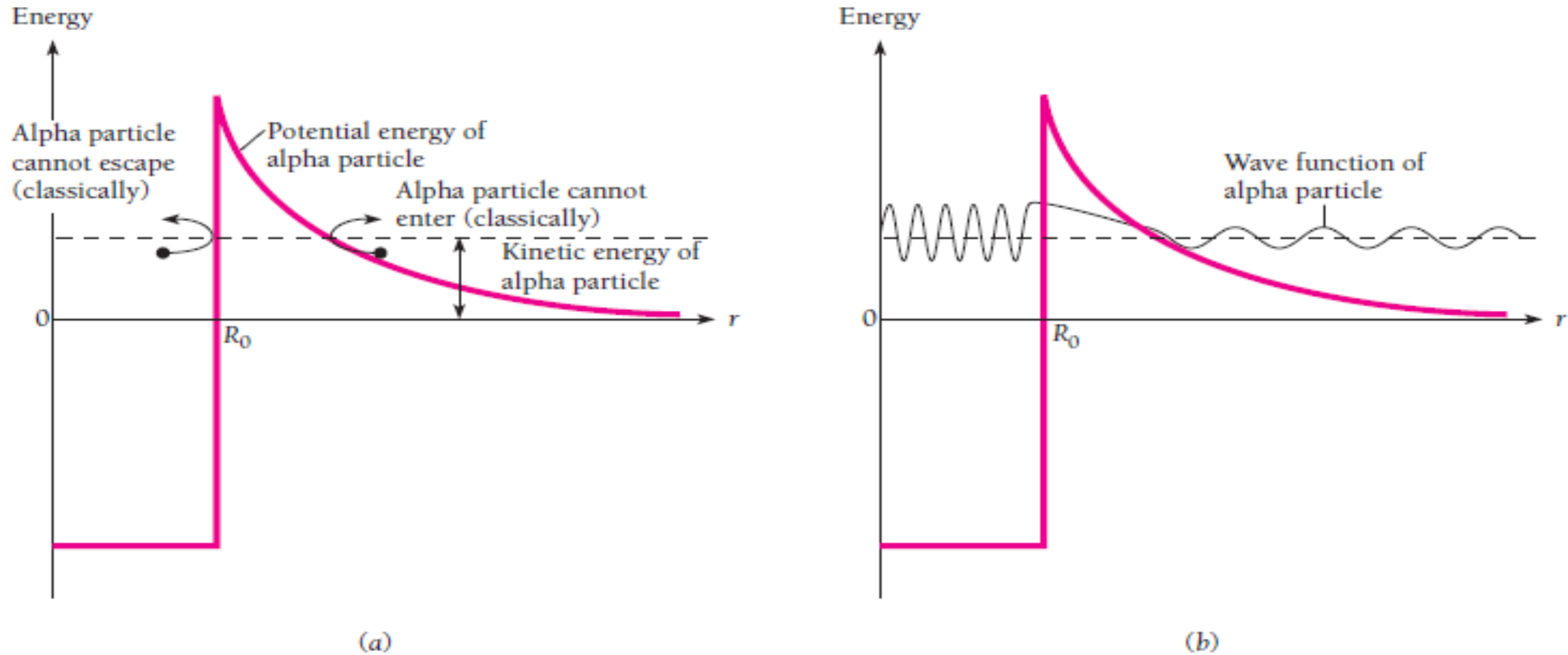
$$Q = K_D + K_{\alpha} = (A/(A-4)) K_{\alpha}$$

$$K_{\alpha} = ((A-4)/A) Q \quad K_D = 4 Q / A$$

A of nearly all alpha emitters exceed 210. Therefore, A-4 is approx. A,  $K_{\alpha} = Q$  and  $K_D$  approx 0.

# Alpha decay cannot be explained by Classical Physics

## Alpha decay can be explained by Tunnel effect of Quantum Mechanics



- (a) In classical physics, an alpha particle whose kinetic energy is less than the height of the potential barrier around a nucleus cannot enter or leave the nucleus, whose radius is  $R_0$ .
- (b) In quantum physics, such an alpha particle can tunnel through the potential barrier with a probability that decreases with the height and thickness of the barrier.

- Figure is a plot of the potential energy  $U$  of an alpha particle as a function of its distance  $r$  from the center of a certain heavy nucleus.
- The height of the potential barrier is about 25 MeV, which is equal to the work that must be done against the repulsive electric force to bring an alpha particle from infinity to a position adjacent to the nucleus but just outside the range of its attractive forces.
- We may therefore regard an alpha particle in such a nucleus as being inside a box whose walls require an energy of 25 MeV to be surmounted.
- Decay alpha particles have energies that range from 4 to 9 MeV, depending on the particular nuclide involved.

## Gamow's Theory of alpha decay

The basic notions of this theory are:

- An alpha particle may exist as an entity within a heavy nucleus.
- Such a particle is in constant motion and is held in the nucleus by a potential barrier.
- There is a small—but definite—likelihood that the particle may tunnel through the barrier.

The decay probability per unit time,  $\lambda$ , can be expressed as

$$\lambda = \nu T$$

$\nu$  : number of times per second an alpha particle within a nucleus strikes the potential barrier

$T$  : probability that the particle will be transmitted through the barrier.

## Gamow's Theory of alpha decay

If we suppose that at any moment only one alpha particle exists as such in a nucleus and that it moves back and forth along a nuclear diameter,

$$\text{(Collision frequency)} \quad \nu = \frac{v}{2 R_0}$$

$v$  : alpha-particle velocity when it eventually leaves the nucleus.

$R_0$  : nuclear radius.

Typical values are:

$$v = 2 \times 10^7 \text{ m/s}$$

$$R_0 = 10^{-14} \text{ m}$$

$$\nu = 10^{21} \text{ s}^{-1}$$

The alpha particle knocks at its confining wall  $10^{21}$  times per second and yet may have to wait an average of as much as  $10^{10}$  y to escape from some nuclei.

## Gamow's Theory of alpha decay

An approximate value of the transmission probability

$$T = e^{-2k_2 L} \quad (2)$$

L: width of the barrier and

$$k_2 = \frac{\sqrt{2m(U - E)}}{\hbar} \quad (3)$$

Eq. (2) is for a rectangular potential barrier.

An alpha particle inside a nucleus is faced with a barrier of varying height.

$$\ln T = -2k_2 L \quad (4)$$

Expressing it as the integral

$$\ln T = -2 \int_0^L k_2(r) dr = -2 \int_{R_0}^R k_2(r) dr \quad (5)$$

$R_0$ : radius of nucleus, R: distance from center at which  $U = E$ .

## Gamow's Theory of alpha decay

For  $r = R$ , (kinetic energy)  $E =$  (potential energy)  $U$ .

As  $r > R$ , the particle permanently escapes the nucleus.

Electric potential energy of  $\alpha$ -particle at the distance  $r$

$$U(r) = \frac{2Ze^2}{4\pi\epsilon_0 r} \quad (6)$$

$Z$ : atomic number of the daughter nucleus.

We therefore have

$$k_2 = \frac{\sqrt{2m(U - E)}}{\hbar} = \left(\frac{2m}{\hbar^2}\right)^{1/2} \left(\frac{2Ze^2}{4\pi\epsilon_0 r} - E\right)^{1/2} \quad (7)$$

Since  $U = E$  when  $r = R$ ,

$$E = \frac{2Ze^2}{4\pi\epsilon_0 R} \quad (8)$$

## Gamow's Theory of alpha decay

$k_2$  can be rewritten as

$$k_2 = \left( \frac{2mE}{\hbar^2} \right)^{1/2} \left( \frac{R}{r} - 1 \right)^{1/2} \quad (9)$$

Hence,

$$\begin{aligned} \ln T &= -2 \int_{R_0}^R k_2(r) dr \\ &= -2 \left( \frac{2mE}{\hbar^2} \right)^{1/2} \int_{R_0}^R \left( \frac{R}{r} - 1 \right)^{1/2} dr \end{aligned} \quad (10)$$

Substituting

$$r = R \cos^2 \theta, \quad dr = -2R \sin \theta \cos \theta d\theta$$

$$\int \left( \frac{R}{r} - 1 \right)^{1/2} dr = -2R \int \sin^2 \theta d\theta$$

$$\ln T = -2 \left( \frac{2mE}{\hbar^2} \right)^{1/2} R \left[ \cos^{-1} \left( \frac{R_0}{R} \right)^{1/2} - \left( \frac{R_0}{R} \right)^{1/2} \left( 1 - \frac{R_0}{R} \right)^{1/2} \right] \quad (11)$$

## Gamow's Theory of alpha decay

Since the potential barrier is relatively wide,  $R \gg R_0$ , and

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\sin\left(\frac{R_0}{R}\right)^{1/2} \approx \left(\frac{R_0}{R}\right)^{1/2}$$

$$\cos\left(\frac{\pi}{2} - \left(\frac{R_0}{R}\right)^{1/2}\right) \approx \left(\frac{R_0}{R}\right)^{1/2}$$

$$\cos^{-1}\left(\frac{R_0}{R}\right)^{1/2} \approx \frac{\pi}{2} - \left(\frac{R_0}{R}\right)^{1/2}$$

and

$$1 - \left(\frac{R_0}{R}\right)^{1/2} \approx 1$$

## Gamow's Theory of alpha decay

$$\ln T = -2 \left( \frac{2mE}{\hbar^2} \right)^{1/2} R \left[ \frac{\pi}{2} - 2 \left( \frac{R_0}{R} \right)^{1/2} \right]$$

Using

$$R = \frac{2Ze^2}{4\pi\epsilon_0 E}$$

we have

$$\ln T = \frac{4e}{\hbar} \left( \frac{m}{\pi\epsilon_0} \right)^{1/2} Z^{1/2} R_0^{1/2} - \frac{e^2}{\hbar\epsilon_0} \left( \frac{m}{2} \right)^{1/2} ZE^{-1/2}$$

Substituting the values of various constants leads to

$$\ln T = 2.97Z^{1/2}R_0^{1/2} - 3.95ZE^{-1/2}$$

$E$  is energy in MeV,  $R_0$  is the nuclear radius in fermis, and  $Z$  is the atomic number of the daughter nucleus.

## Gamow's Theory of alpha decay

Since

$$\log_{10} A = (\log_{10} e)(\ln A) = 0.4343 \ln A$$

$$\log_{10} T = 1.29Z^{1/2}R_0^{1/2} - 1.72ZE^{-1/2}$$

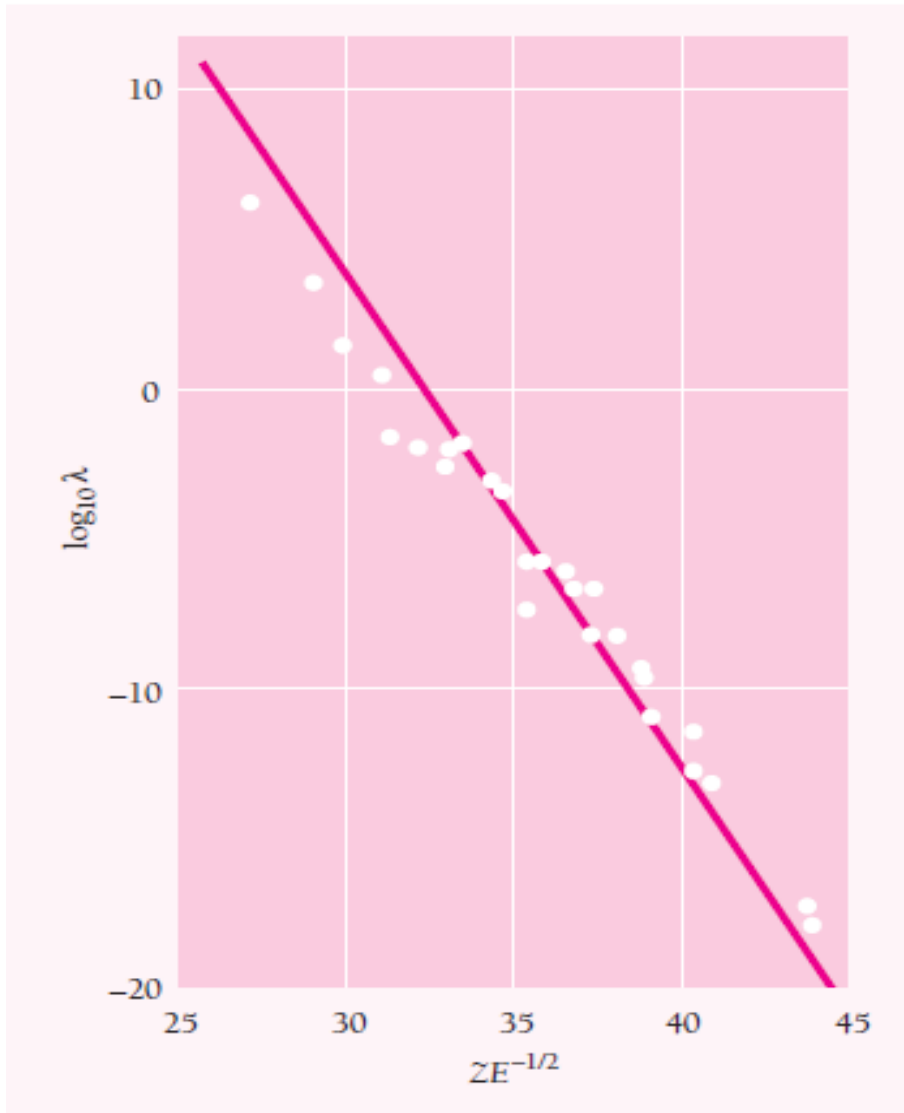
As defined, decay constant

$$\lambda = \nu T = \frac{\nu}{2R_0} T$$

Taking  $\log_{10}$  on both sides and substituting for T

$$\log_{10} \lambda = \log_{10} \left( \frac{\nu}{2R_0} \right) + 1.29Z^{1/2}R_0^{1/2} - 1.72ZE^{-1/2}$$

Experimental verification of Gamow's Theory  $\log_{10} \lambda = \log_{10} \left( \frac{v}{2R_0} \right) + 1.29Z^{1/2}R_0^{1/2} - 1.72ZE^{-1/2}$



- The line fitted to the experimental data has 1.72 slope as predicted by theory.
- The nuclear radius,  $R_0$ , determined from the position of the line is just about what is obtained from nuclear scattering experiments.
- Theory predicts that the decay constant, and hence the half-life, should vary strongly with the alpha-particle energy  $E$ . This is indeed the case.
- The slowest decay is that of  $^{232}\text{Th}_{90}$ , whose half-life is  $1.3 \times 10^{10}$  y and the alpha particle energy is 4.05 MeV, and the fastest decay is that of  $^{212}\text{Po}_{84}$ , whose half-life is  $3.0 \times 10^7$  s and the alpha particle energy is 8.95 MeV.

## Reference Books:

- Concepts of Modern Physics, Arthur Beiser, 2002, McGraw-Hill.
- Introduction to Nuclear and Particle Physics, V. K. Mittal, R. C. Verma, S. C. Gupta, 3<sup>rd</sup> Edition, PHI Learning Private Limited.

Thanks