Revision :- "Fourier Series"

Any periodic non sinusoidal function can be expressed ~~as~~ or decomposed into a fundamental and its harmonics which is a series of Sines and cosines of an angle and its multiples of the form

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots + a_n \cos nx + \cdots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots + b_n \sin nx + \cdots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

is called the Fourier series, where $a_0, a_1, a_2, \cdots a_n, \cdots b_1, b_2, b_3, \cdots b_n$ are constants.

⇒ a_0 represents constant term

⇒ a_1 & b_1 gives the component at fundament freq.

⇒ $a_2, a_3 \cdots b_2, b_3 \cdots$ gives components of harmonics ie multiples of the fundamental frequency

Note :- Fourier series is the series which is applicable for both continuous and discontinuous types of functions

* (B)

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(5) The Fourier series is used to represent a periodic function by a discrete sum of complex exponentials, while the Fourier transform is then used to represent a general non periodic function by a continuous superposition or integral of complex exponentials.

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Thus we see examples (1) to (5), we are

Any periodic function can be written as a fourier series. Any function can be
decomposed into freq components. This is the change of variable from the time domain into

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* { Integral: Fourier, Laplace }

Integral Transforms

⇒ Integral Transforms are used in the solution of partial differential equations

⇒ There are many Transforms; The choice of Transform depends to be used as solution of partial differential depends on (a) nature of boundary conditions
(b) facility with which the Transform $F(s)$ can be converted to give $f(x)$

The integral transform "$F(s)$" of a function $f(x)$ with the kernel $K(s,x)$" is defined as

$$I[f(x)] = F(s) = \int_a^b f(x)\, K(s,x)\, dx \quad \to (1)$$

s is variable, $K(s,x)$ is a "pair function".

Transformation is nothing but a kind of Mapping. for eg.

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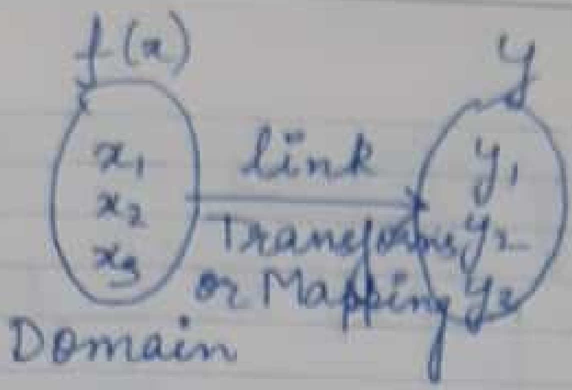
$|\psi|^2$ $\psi^* \psi$



eq① represents the mapping between F(s) and f(x).

examples :-

① If $k(s,x) = e^{isx}$

$$F[f(x)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

Such a Transform is known as fourier complex transform.

Now, it's Inverse $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$

(Inversion formula) Or Inverse Fourier Transform.

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(2) Physical Significance of

$I(f(x))$ or $F(f(x))$ or $L(f(x))$

$I \to$ Integral, $F \to$ Fourier, $L \to$ Laplace.

It is basically a kind of Mappin which establishes the relation between two spaces (eg. S and

(2.) If kernel $k(S,x) = \sin Sx$

$$F_s[f(x)] = F_s(S) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin Sx \, dx$$

F_s = Fourier Sin transform

Inverse Fourier sin transform is

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_s(s) \sin Sx \, dS$$

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$e^{ix} = i\sin x + \cos x$

$\cos\theta + i\sin\theta$

(3.) If Kernel $K(s,x) = \cos sx$

$$F_c[f(x)] = F_c(s) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\, dx$$

F_c = Fourier cosine transform

⇒ Inverse Fourier Cos Transform will be.

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_c(s)\cos sx\, ds.$$

(4.) If kernel $K(s,x) = e^{-sx}$

I → L

and hence, $L(f(x)) = L(s) = \int_0^\infty f(x)\, e^{-sx}\, dx$

The mapping will be like

f(x)	L(s)
$f(x_1)$	$L(s_1)$
$f(x_2)$	$L(s_2)$

$\sqrt{\frac{2}{\pi}} + \sqrt{\frac{2}{\pi}}$

$2\sqrt{\frac{2}{\pi}}$

$\frac{\sqrt{2}\times\sqrt{2}\times\sqrt{2}}{\sqrt{\pi}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $\quad \frac{4}{\sqrt{2\pi}}$

$\frac{(\sqrt{2})^3}{\sqrt{\pi}}$

$\sqrt{a}\times\sqrt{a}$

$(a)^{1/2}\times(a)$

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(5) "Hankel Transform" with the kernel $(k, s) = x J_n(sx)$

$J_n(sx)$ is a bessel function

$I \rightarrow H$

$$H[f(x)] = F(s) = \int_0^\infty f(x) \cdot x J_n(sx) dx$$

** Inverse Hankel Transform will be

(Not to be done Class)

$$f(x) = \int_0^\infty$$

(6) Hilbert Transform with the kernel $k(s, x) = \frac{1}{s-x}$

$$F(s) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(x)}{s-x} dx$$

(7) Mellin transform with the kernel

THINGS TO DO $k(s, x) = x^{s-1}$

$$M[f(x)] = F(s) = \int_0^\infty f(x) \cdot x^{s-1} dx$$

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If we see examples 1 to 5, we are seeing that $K(x,s)$ (Kernel or pair function) is always periodic i.e. e^{-sx}, e^{isx}, $\cos x$, $\sin x$, $J_n(sx)$.

Fourier Integral Theorem

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

↳ Fourier Series of non sinusoidal Periodic function $f(x)$

dirichlet's conditions for a fourier series.

If the function $f(x)$ for the interval $(-\pi, \pi)$

1) is single valued
2) is bounded
3) has at most a finite no. of maxima & minima.
4) has only a finite no. of discontinuities
5) is $f(x+2\pi) = f(x)$ for values of x outside $[-\pi, \pi]$, then

~~$t-x)\,dt$~~

$$S_p(x) = \frac{a_0}{2} + \sum_{n=1}^{p} a_n \cos nx + \sum_{n=1}^{p} b_n \sin nx$$

converges to $f(x)$ as $p \to \infty$ at values of x for which $f(x)$ is continuous and the sum of the series is equal to

$$\frac{1}{2}\left[f(x+0) + f(x-0)\right]$$ at points of discontinuity.

Theorem:

<u>Statement</u>:- If $f(x)$ satisfies the following conditions:

(i) $f(x)$ satisfies the "Drichlet conditions in every interval $-l \le x \le l$

(ii) $\int_{-\infty}^{\infty} |f(x)|\,dx$ converges i.e, $f(x)$ is absolutely integrable in the interval $-\infty < x < \infty$, then

$$f(x) = \frac{1}{2\pi} \int_{p=-\infty}^{\infty} \int_{t=-\infty}^{\infty} f(t) \cos p(t-x)\, dp\, dt$$

The integral on R.H.S is called Fourier Integral or Fourier Integral expansion of $f(x)$

Proof:- Let us consider a function Satisfying Dritchlet's conditions in every interval $(-c, c)$ however large. Also suppose that

$$\int_{-\infty}^{\infty} |f(x)|\, dx \text{ is convergent. } -(1)$$

Then in the Interval $(-c, c)$,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{c} + b_n \sin \frac{n\pi x}{c} \right] \rightarrow (2)$$

where $a_n = \frac{1}{c} \int_{-c}^{c} f(t) \cos \frac{n\pi t}{c}\, dt \quad , n = 0, 1, 2,$

$b_n = \frac{1}{c} \int_{-c}^{c} f(t) \sin \frac{n\pi t}{c}\, dt \quad , n = 0, 1, 2$

$a_0 = \frac{1}{2c} \int_{-c}^{c} f(t)\, dt$

Putting the values of a_n and b_n in (2)

$$f(x) = \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{c}\sum_{n=1}^{\infty}\int_{-c}^{c} f(t)\left\{\cos\frac{n\pi t}{c}\cos\frac{n\pi x}{c} + \sin\frac{n\pi t}{c}\sin\frac{n\pi x}{c}\right\}dt$$

$$= \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{c}\sum_{n=1}^{\infty}\int_{-c}^{c} f(t)\cos\frac{n\pi}{c}(t-x)\,dt$$

$$\left[\cos A\cos B + \sin A\sin B = \cos(A-B)\right]$$

making use of the fact that $f(x)$ is uniformly convergent in the closed interval $-c \le x \le c$, we get

$$f(x) = \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{c}\int_{-c}^{c} f(t)\left[\sum_{n=1}^{\infty}\cos\frac{n\pi(t-x)}{c}\right]dt$$

Making use of

$$= \frac{1}{2c}\int_{-c}^{c} f(t)\,dt\left[1 + \lim_{n\to\infty}\sum_{r=1}^{n} 2\cos r\pi\frac{(t-x)}{c}\right]dt$$

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$$= \frac{1}{2c}\int_{-c}^{c} f(t)\left[1+\lim_{n\to\infty}\sum_{r=1}^{n}\left\{\cos\frac{r\pi(t-x)}{c}+\cos\frac{-r\pi(t-x)}{c}\right\}\right]dt$$

$\cos(-\theta)=\cos\theta$

$$= \frac{1}{2c}\int_{-c}^{c} f(t)\left[1+\lim_{n\to\infty}\sum_{r=-n}^{n}\cos\frac{r\pi(t-x)}{c}\right]dt$$

$$= \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{2\pi}\int_{-c}^{c} f(t)\left[\lim_{n\to\infty}\sum_{r=-n}^{n}\frac{1}{c/\pi}\cos\left[\frac{r}{c/\pi}\right]\right.$$

Making use of definition of integral as a limit of sum, we get

$$f(x) = \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{2\pi}\int_{-c}^{c} f(t)\left[\int_{-\infty}^{\infty}\cos u(t-x)\right.$$

By putting

$$\boxed{u = \frac{r}{c/\pi}}$$

$$du = \frac{dr}{c/\pi}$$

$$\frac{dr}{c/\pi}$$

$$dr = \left(\frac{c}{\pi}\right)\times du$$

$\cancel{\frac{x}{2\pi}}$ $\frac{x\pi}{c}$

$$f(x) = \frac{1}{2c}\int_{-c}^{c} f(t)\,dt + \frac{1}{2\pi}\int_{-c}^{c} f(t)\,dt \int_{-\infty}^{\infty} \cos u(t-x)\,du$$

Making $c \to \infty$ and using the fact ①, we get

$$f(x) = \cancel{0} + \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\,dt \int_{-\infty}^{\infty} \cos u(t-x)\,du$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\cos u(t-x)\,dt\,du$$

Finally if $-\infty < x < \infty$, then

$$f(x) = \frac{1}{2\pi}\int_{p=-\infty}^{\infty}\int_{t=-\infty}^{\infty} f(t)\cos p(t-x)\,dp\,dt$$

$(t-x)\,]\,dt$

$\int \cos u(t-x)\frac{c}{\pi}\,du$

$du\,]\,dt$

$$\int_{-\infty}^{\infty} f(x)\,dx = 0$$

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Different forms of Fourier Integral Formula

(i) $$f(x) = \frac{1}{\pi}\int_{p=0}^{\infty}\int_{t=-\infty}^{\infty} f(t)\cos p(t-x)\,dp\,dt$$

Proof: By Fourier's integral formula

$$f(x) = \frac{1}{2\pi}\int_{p=-\infty}^{\infty}\cos p(t-x)\,dp\int_{t=-\infty}^{\infty} f(t)\,dt$$

$$= \frac{2}{2\pi}\int_{0}^{\infty}\cos p(t-x)\,dp\int_{-\infty}^{\infty} f(t)\,dt$$

$$\left[\because \int_{-a}^{a} f(p)\,dp = 2\int_{0}^{a} f(p)\,dp \text{ if } f(p) = f(-p)\right]$$ for even function

$$\boxed{f(x) = \frac{1}{\pi}\int_{0}^{\infty}\int_{-\infty}^{\infty} f(t)\cos p(t-x)\,dp\,dt}$$

(ii) Cosine form
complex representation of fourier Transform

Now

$$f(x) = \frac{1}{\pi}\int_{0}^{\infty}\int_{-\infty}^{\infty} f(t) \cos p(t-x)\, dp\, dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, dt \int_{-\infty}^{\infty} \cos p(t-x)\, dp \quad (1)$$

If function $f(t)$ to be an odd function

For that $\int_{-a}^{a} f(t)\, dt = 0 \quad -(2)$

and if we take sin func (odd fuc)

then $\int_{-\infty}^{\infty} \sin p(t-x)\, dp = 0 \quad -(3)$

So, we can write

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, dt \int_{-\infty}^{\infty} \sin p(t-x)\, dp = 0 \quad -(4)$$

multiply (4) by i

$$\frac{i}{2\pi}\int_{-\infty}^{\infty} f(t)\,dt \int_{-\infty}^{\infty} \sin p(t-x)\,dp = 0 \times i \quad — (5)$$

Adding (1) & (5)

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\,dt \int_{-\infty}^{\infty} \cos p(t-x)\,dp + \frac{i}{2\pi}\int_{-\infty}^{\infty} f(t)\,dt$$

$$\boxed{f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\,dt \int_{-\infty}^{\infty} e^{ip(t-x)}\,dp}$$

↳ complex representation of fourier tranform.

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\,e^{ipt}\,dt \int_{-\infty}^{\infty} e^{-ipx}\,dp$$

Let $p = s$

$$\text{Let } f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)\,e^{ist}\,e^{-isx}\,dt\,ds$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,e^{ist}\,dt = F(s)$$

⇓

This is called Fourier Transform of func $f(x)$

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and Inverse Fourier Transform will be $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s)\, e^{-isx}\, ds$

$$\boxed{dt \int_{-\infty}^{\infty} \sin p(t-x)\, dp}$$

Sin Transform:⇒

$$F[f(t)] = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{ist}\, dt$$

$$\boxed{\text{Or } F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx}\, dx}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)[\cos st + i \sin st)\, dt$$

Taking imaginary part

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \sin st\, dt$$

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when $f(t)$ is

$$= \frac{1}{\sqrt{2\pi}} 2 \int_0^\infty f(t) \sin st \, dt$$

$$F_s(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(t) \sin st \, dt$$

Inverse fourier Transform of Sine Transform

$$f(t) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_s(s) \sin st \, dt$$

Now, taking Real part

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \cos st \, dt$$

$$F_c(s) = \frac{2}{\sqrt{2\pi}} \int_0^\infty f(t) \cos st \, dt$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty f(t) \cos st \, dt$$

Inverse Fourier Transform will be

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$$f(t) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(s) \cos st \, ds$$

Properties

1) Linear property

Suppose we have two fourier Transforms $F_1(S)$ and $F_2(S)$ then by linear property, we can add them up

$$F_1(s) + F_2(s)$$

or

2) change of Scalar Property

(a) suppose $F(s)$ is a Fourier transform of $F(t)$. Now, we know that

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{ist}\, dt$$

If c_1 & c_2 are arbitrary constants, then

$$F\{c_1 F(\cancel{t}) \pm c_2 G(x)\} = c_1 F\{F(x)\} + c_2 F\{G(x)\}$$

$t = x$

Proof - we have

$$F\{c_1 F(x) \pm c_2 G(x)\}$$

$$= \int_{-\infty}^{\infty} e^{ipx} [c_1 F(x) \pm c_2 G(x)]\, dx$$

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$$= \int_{-\infty}^{\infty} [C_1 e^{ipx} F(x) \pm C_2 e^{ipx} G(x)] \, dx$$

$$= \int_{-\infty}^{\infty} [C_1 e^{ipx} F(x) \pm C_2 e^{ipx} G(x)] \, dx$$

$$= \int_{-\infty}^{\infty} C_1 e^{ipx} F(x) \, dx \pm \int_{-\infty}^{\infty} C_2 e^{ipx} G(x) \, d$$

$$= C_1 F\{F(x)\} \pm C_2 F\{G(x)\}$$

__change of scale Property continued:-__

$\Rightarrow$ then $\frac{1}{a} f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(at)$

we have

$$F\{f(t)\} = \int_{-\infty}^{\infty} e^{ist} F(t) \, dt = F(s)$$

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$$\therefore F\{f(at)\} = \int_{-\infty}^{\infty} e^{ist} f(at)\,dt$$

$$\cancel{= \int_{-\infty}^{\infty} e^{\frac{-ist}{a}} F(}$$

Put $at = x$

$$= \int_{-\infty}^{\infty} e^{\frac{isx}{a}} f(x) \frac{dx}{a}$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} e^{\left(\frac{is}{a}\right)x} f(x)\,dx$$

$$= \frac{1}{a} F\left(\frac{s}{a}\right) \quad \text{by definition}$$

Known $F(s) = \int_{-\infty}^{\infty} e^{isx} f(x)\,dx$

$\hookrightarrow$ definition

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$F_s(as)$

(b.) If $\cancel{f_s(s)}$ is the Fourier sine transform of $F(t)$, then Fourier sine Transform of $f\left(\frac{t}{a}\right)$ is $a f_s(as)$.

⇒ Basically you have to find $F_s\left(f\left(\frac{t}{a}\right)\right)$

$$\cancel{F_s\left\{F\left(\frac{x}{a}\right)\right\} = \int_0^{\infty} F\left(\frac{x}{a}\right) \sin px \, dx}$$

$$\cancel{= \int_0^{\infty} F}$$

$$F_s\left\{f\left(\frac{t}{a}\right)\right\} = \int_0^{\infty} f\left(\frac{t}{a}\right) \sin st \, dt$$

Put $\frac{t}{a} = x$ ⇒ $dt = a\,dx$

$$= \int_0^{\infty} f(x) \sin(sax) \, a \, dx$$

$$= \int_0^{\infty} a f(x) \sin(sax) \, dx$$

$$= a F_s(as).$$

Shifting property :-

If $Ff(s)$ is the Fourier transform of $f(t)$ then, $e^{ias}F(s)$ is the Fourier transform of $f(t-a)$

$$F\{f(t-a)\} = \int_{-\infty}^{\infty} e^{ist} f(t-a)\,dt$$

Put

$t-a = x$

$dt = dx$

$$= \int_{-\infty}^{\infty} e^{is(x+a)} f(x)\,dx$$

$$= \int_{-\infty}^{\infty} e^{isx} e^{isa} f(x)\,dx$$

$$= e^{isa} \int_{-\infty}^{\infty} e^{isx} f(x)\,dx$$

$$= e^{isa} Ff(s).$$

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Modulation Theorem :-

Theorem:- If $f(t)$ has the Fourier transform of $F(s)$, then $f(t)\cos at$ has the Fourier transform

$$\frac{1}{2}F(s-a)+\frac{1}{2}F(s+a)$$

We know that

$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$$

$$F\{f(t)\cos at\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ist} f(t)\cos at\,dt$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{ist} f(t)\left(\frac{e^{iat}+e^{-iat}}{2}\right)dt$$

$$= \frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[f(t)e^{ist}e^{iat}\,dt + f(t)e^{ist}e^{-iat}\,dt\right]$$

$$= \frac{1}{2\sqrt{2\pi}}\left\{\int_{-\infty}^{\infty} f(t)e^{it(s+a)}\,dt + \int_{-\infty}^{\infty} f(t)e^{it(s-a)}\,dt\right\}$$

IMPORTANT

SATURDAY 27

$$= \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{it(a+s)} dt + \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{it(a-s)} dt$$

$$= \frac{1}{2} F(s+a) + \frac{1}{2} F(s-a)$$

{By using Shifting Property}

Find $F_s(f(x) \sin ax)$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin ax \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \times \frac{1}{2} \int_0^{\infty} f(x) \, 2 \sin sx \sin ax \, dx$$

$$= \sqrt{\frac{2}{\pi}} \times \frac{1}{2} \int_0^{\infty} f(x) \left[\cos[s-a]x - \cos[s+a]x \right] dx$$

$\cos(A-B) - \cos(A+B)$
$= 2 \sin A \sin B$

THINGS TO DO

$$= \frac{1}{2} \left[F_c(s-a) - F_c(s+a) \right]$$

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JANUARY

28 SUNDAY

Find $F_c[f(x)\cos ax]$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\sin(a+b) = \sin a \cos b$

$$= \sqrt{\frac{2}{\pi}}\frac{2}{2}\int_0^\infty f(x)\cos ax \cos sx\,dx$$

$$= \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos(ax+sx) + \cos(sx-ax)$$

$$= \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_0^\infty \left[f(x)\cos x(s+a) + \cos x(s-a)\right]dx$$

$$= \frac{1}{2}\left[\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos x(s+a)\,dx + \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos x\right]$$

$$= \frac{1}{2}\left[F_c(s+a) + F(s-a)\right]$$

THINGS TO DO

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$\sin(a-b) = \sin a \cos b - \sin b \cos a$

Solve $F_c(f(x) \sin ax)$

sinb cosa

$$F_c(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \sin ax \, dx$$

$$= \sqrt{\frac{2}{\pi}} \times \frac{1}{2} \int_0^\infty f(x)\, 2 \cos sx \sin ax \, dx$$

dx

$$= \frac{1}{2}\left[\sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(sx+ax)\, dx - \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(sx-ax)\, dx\right]$$

(s-a) dx

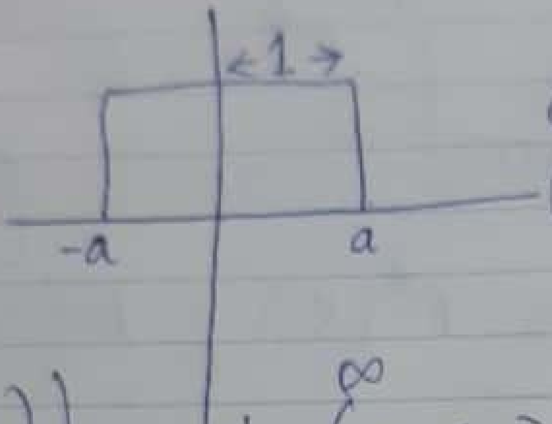
$$= \frac{1}{2}\left[F_s(s+a) - F_s(s-a)\right]$$

By shifting Property

03 SATURDAY

Find the fourier transform of function

$$f(x) = 1 \quad |x| \le a$$
$$= 0 \quad |x| > a$$



graphical representation

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} 1 \cdot f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left. \frac{e^{isx}}{is} \right|_{-a}^{a}$$

$$dax = a$$

2018

IMPORTANT

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04

SUNDAY

$$\frac{1}{\sqrt{2\pi}} \frac{e^{isa} - e^{-isa}}{is} \times \frac{2}{2}$$

$$\sqrt{\frac{2}{\pi}} \frac{e^{isa} - e^{-isa}}{2i} \times \frac{1}{s}$$

$$= \sqrt{\frac{2}{\pi}} \frac{\sin sa}{s}$$

$$= \sqrt{\frac{2}{\pi}} \frac{\sin sa}{s} \times \frac{a}{a}$$

$$\sqrt{\frac{2}{\pi}}\, a \frac{\sin sa}{sa}.$$

Now, Taking limit

$$\sqrt{\frac{2}{\pi}}\, a \lim_{s \to 0} \frac{\sin sa}{sa}$$

→ Indeterminant form or $\frac{0}{0}$ form

THINGS TO DO

In order to remove singularity apply L'Hospital rule

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$$\sqrt{\frac{2}{\pi}}\ \cancel{\alpha} \cdot \frac{a\cos sa}{\cancel{\alpha}}$$

$$= \sqrt{\frac{2}{\pi}}\ a\cos a$$

Fourier transform of $f(x) = \frac{1}{\varepsilon}$ $|x| \le \varepsilon$
$= 0$ $|x| > \varepsilon$

→ from copy

L'hospital rule

If we have an indeterminant form 0/0 or ∞/∞ all we need to do is differentiate the numerator and differentiate the denominator and then take the limit

THINGS TO DO

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IMPORTANT

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TUESDAY 06

$$\frac{1}{\sqrt{2\pi}}\int_{-\epsilon}^{\epsilon} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\epsilon} \frac{e^{isx}}{\epsilon}\, dx$$

$$= \frac{1}{\sqrt{2\pi}}\frac{1}{\epsilon}\frac{e^{isx}}{is}\Bigg|_{-\epsilon}^{\epsilon}$$

$$= \frac{1}{\sqrt{2\pi}}\frac{e^{is\epsilon} - e^{-is\epsilon}}{\epsilon\, is} \times \frac{2}{2}$$

$$\frac{1}{\sqrt{2\pi}}\frac{2}{\epsilon s}\cdot\frac{e^{is\epsilon} - e^{-is\epsilon}}{2i}$$

$$= \sqrt{\frac{2}{\pi}}\,\frac{2}{\epsilon s}\sin s\epsilon$$

$$= \sqrt{\frac{2}{\pi}}\,\frac{\sin s\epsilon}{s\epsilon}$$

$$\sqrt{\frac{2}{\pi}}\lim_{s\to 0}\frac{\sin s\epsilon}{s\epsilon}$$

$$\sqrt{\frac{2}{\pi}}\frac{\cancel{\epsilon}\cos s\epsilon}{\cancel{\epsilon}} = \sqrt{\frac{2}{\pi}}\cos s\epsilon$$

THINGS TO DO

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TBD

Q Find the Fourier transform of

$$f(x) = \begin{cases} x, & |x| \le a \\ 0 & |x| > a. \end{cases}$$

$$F(f(x)) = \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

$$= \int_{-a}^{a} x e^{isx} dx.$$

$=$

Q $F_s(\alpha) = \frac{e^{-ax}}{x}$ $f(x)$.

TBD

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{x} \sin \alpha x \, dx$$

$$F_s'(\alpha) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{e^{-ax}}{\cancel{x}} \cancel{x} \cos \alpha x \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos \alpha x \, dx.$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos \alpha x \, dx$$

$$= \mathrm{Re} \int_0^{\infty} e^{-ax} e^{i\alpha x} dx$$

$$= \mathrm{Re} \int_0^{\infty} e^{-(a-i\alpha)x} dx$$

$$= \mathrm{Re} \left[\frac{e^{-(a-i\alpha)x}}{-(a-i\alpha)} \right]$$

$$= \mathrm{Re} \left[\frac{1}{a-i\alpha} \right]$$

$$= \mathrm{Re} \left[\frac{a+i\alpha}{(a-i\alpha)(a+i\alpha)} \right]$$

$$= \mathrm{Re} \left[\frac{a+i\alpha}{a^2 - i^2 \alpha^2} \right]$$

THINGS TO DO

$$= \frac{a}{a^2+\alpha^2}.$$

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$$F_s'(\alpha) = \frac{a}{a^2+\alpha^2} \times \sqrt{\frac{2}{\pi}}$$

$$F_s(\alpha) = a\int_0^\infty \frac{d\alpha}{a^2+\alpha^2} \times \sqrt{\frac{2}{\pi}}$$

$$= \sqrt{\frac{2}{\pi}}\ \cancel{\alpha} \cdot \frac{1}{\cancel{\alpha}} \tan^{-1}\frac{\alpha}{a} + C$$

$$\text{y } \alpha = 0,\ F_s(\alpha) = 0$$

$$C = 0$$

$$F_s(\alpha) = \sqrt{\frac{2}{\pi}} \tan^{-1}\frac{\alpha}{a}.$$

$$F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{ist}\, dt$$

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} e^{t} e^{ist}\, dt + \int_0^\infty e^{-t} e^{ist}\, dt\right]$$

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IMPORTANT

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10
SATURDAY

$|t| = t \quad t > 0$

$|t| = -t \quad t < 0$

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} e^{(1+is)t}\,dt + \int_{0}^{\infty} e^{-(-is+1)t}\,dt\right]$$

$$\frac{1}{\sqrt{2\pi}}\left[\frac{e^{(1+is)t}}{1+is}\Bigg|_{-\infty}^{0} + \frac{e^{-(1-is)}}{(1-is)}\Bigg|_{0}^{\infty}\right.$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{1}{1+is} + \frac{1}{1-is}\right]$$

$$\frac{2}{1+s^2} \times \frac{1}{\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}} \times \frac{1}{1+s^2}$$

Q:- Prove that Fourier transform of a gaussian function is itself a gaussian function

$$f(x) = Ne^{-\alpha x^2}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Ne^{-\alpha x^2} e^{isx} dx$$

$$= \frac{N}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha x^2 + isx} dx$$

$$\frac{N}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha\left(x^2 - \frac{is}{\alpha}x\right)} dx$$

$$= \frac{N}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha\left(x^2 - \frac{isx}{\alpha} + \left(\frac{is}{2\alpha}\right)^2 - \left(\frac{is}{2\alpha}\right)^2}$$

$$= \frac{N}{\sqrt{2\pi}} e^{-\frac{s^2}{4\alpha^2}} \int_{-\infty}^{\infty} e^{-\alpha\left(x - \frac{is}{2\alpha}\right)^2} dx$$

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Let $x - \frac{is}{2\alpha} = y$

$$F(s) = \frac{N}{\sqrt{2\pi}} e^{-\frac{s^2}{4\alpha^2}} \int_{-\infty}^{\infty} e^{-\alpha y^2} dy$$

$e^{-\alpha y^2} \rightarrow$ even function ✓

$$F(s) = \frac{N}{\sqrt{2\pi}} \times 2 \times e^{-\frac{s^2}{4\alpha^2}} \int_0^{\infty} e^{-\alpha y^2} dy. \checkmark$$

$\cancel{e^{-\alpha t - \frac{i^2 s^2}{4\alpha^2}}}$

Put $\sqrt{\alpha}\, y = t$

$dy = \frac{dt}{\sqrt{\alpha}}$ ✓

$$= \frac{N}{\sqrt{2\pi}} e^{-\frac{s^2}{4\alpha^2}} \times \frac{2}{\sqrt{\alpha}} \int_0^{\infty} e^{-t^2} dt$$

Now put $t^2 = x$

$2t\,dt = dx.$

$dt = \frac{dx}{2\sqrt{x}}.$

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$$= \frac{N}{\sqrt{2\pi}} e^{-\frac{s^2}{4\alpha^2}} \times \frac{2}{\sqrt{\alpha}} \times \frac{1}{2} \int_0^{\infty} e^{-x} x^{-1/2} dx.$$

$$\frac{N}{\sqrt{2\pi}}\, e^{-\frac{s^2}{4\alpha^2}} \times \frac{\cancel{2}}{\cancel{2}\sqrt{\alpha}}$$

$$\frac{N}{\sqrt{2\pi\alpha}}\, e^{-\frac{s^2}{4\alpha^2}} \int_0^\infty e^{-x} x^{-1/2}\, dx$$

$n-1 = -1/2$
$n = 1-1/2$
$n = 1/2$

Now, By definition, gamma function is

$$\int e^{-x} x^{n-1}\, dx = \Gamma n$$

$n-1 = -1/2$

$$\frac{N}{\sqrt{2\pi\alpha}}\, e^{-\frac{s^2}{4\alpha^2}} \times \Gamma\tfrac{1}{2} \checkmark =$$

and also $\Gamma n = (n-1)!$

$$\Gamma\tfrac{1}{2} = \left(\frac{1}{2} - 1\right)!$$

$$= \frac{N}{\sqrt{2\pi\alpha}} \times \sqrt{\pi}\, e^{-\frac{s^2}{4\alpha^2}}$$

THINGS TO DO

$$= \frac{N}{\sqrt{2\alpha}}\, e^{-\frac{s^2}{4\alpha^2}}$$

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Q Find the Fourier transform of $x f(x)$ where $f(x) = 1-x^2$ if $|x| \le 1$

$= 0$ if $|x| > 1$

Q:- Find the sine and cosine Transform of $f(x) = e^{-ax}$ $a>0$

M(U) TBD

$$F_s(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \underset{I}{e^{-ax}} \, \underset{II}{\sin sx} \, dx$$

$$= \sqrt{\frac{2}{\pi}} \cdot \left[\frac{s}{s^2+a^2} \right]$$

$$(f(x)] = \frac{a}{s^2+a^2} \sqrt{\frac{2}{\pi}}$$

Now, find the value of

(1) $\int_0^{\infty} \frac{\cos \alpha x}{a^2+\alpha^2} d\alpha$ (2) $\int_0^{\infty} \frac{\alpha \sin \alpha x \, d\alpha}{a^2+\alpha^2}$

Now applying Inverse cosine Transform.

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_c(\alpha) \cos \alpha x \, d\alpha.$$

$$e^{-ax} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{a}{a^2+\alpha^2} \cos \alpha x \, d\alpha.$$

$$= \frac{2}{\pi} a \int_0^{\infty} \frac{\cos \alpha x}{a^2+\alpha^2} d\alpha$$

$$\boxed{\frac{\pi}{2a} e^{-ax} = \int_0^{\infty} \frac{\cos \alpha x}{a^2+\alpha^2} d\alpha}$$

Now apply Inverse Sine Transform

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_s(\alpha) \sin \alpha x \, d\alpha$$

$$e^{-ax} = \sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{2}{\pi}} \frac{\alpha}{\alpha^2 + a^2} \sin \alpha x \, d\alpha$$

$$= \sqrt{\frac{2}{\pi}} \times \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\alpha \sin \alpha x}{a^2 + \alpha^2} \, d\alpha$$

$$\frac{\pi e^{-ax}}{2} = \int_0^\infty \frac{\alpha \sin \alpha x}{a^2 + \alpha^2} \, d\alpha.$$

Find the Fourier Transform of f(x)

$$f(x) = 1 \quad |x| \le a$$
$$= 0 \quad |x| > a$$

evaluate the integrals.

$$\int_{-\infty}^{\infty} \frac{\sin a\alpha \cos \alpha x}{\alpha} \, d\alpha$$

$$\int_{-\infty}^{\infty} \frac{\sin \alpha}{\alpha} \, d\alpha$$

$$\int_0^\infty \frac{\sin a\alpha \cos \alpha x}{\alpha} \, d\alpha$$

$$F(\alpha) = \sqrt{\frac{2}{\pi}} \frac{\sin a\alpha}{\alpha}.$$

Inverse fourier transform

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha)\, e^{-i\alpha x}\, d\alpha$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}}\, e^{-i\alpha x} \frac{\sin a\alpha}{\alpha}\, d\alpha$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin a\alpha}{\alpha} (\cos \alpha x - i \sin \alpha x)\, d\alpha$$

$$\pi f(x) = \int_{-\infty}^{\infty} \left(\frac{\sin a\alpha \cos \alpha x}{\alpha} - i \frac{\sin a\alpha \sin \alpha x}{\alpha} \right.$$

in condition $|x| \leq 1$ the $f(x) = 1$, so we take only real part, neglecting imaginary term

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$$= \int_{-\infty}^{\infty} \frac{\sin a\alpha \cos \alpha x}{\alpha} \, d\alpha = \pi f(x) \quad (1)$$

$$\text{So } = \pi \; \text{if } |x| \le a$$
$$= 0 \; \text{if } |x| > a$$

if $x = 0$

$$\int_{-\infty}^{\infty} \frac{\sin \alpha a}{\alpha} \, d\alpha = \pi$$

→ This is an even function

$$\Rightarrow 2 \int_{0}^{\infty} \frac{\sin \alpha a}{\alpha} \, d\alpha = \pi \quad \text{Put } a = 1$$

$$d\alpha \Rightarrow \int_{0}^{\infty} \frac{\sin \alpha}{\alpha} \, d\alpha = \frac{\pi}{2} \quad (2)$$

Now in (1).

$$\int_{-\infty}^{\infty} \frac{\sin a\alpha \cos \alpha x}{\alpha} \, d\alpha = 2 \int_{0}^{\infty} \frac{\sin a\alpha \cos \alpha x}{\alpha} \, d\alpha$$

even even = Total even

FEBRUARY

19 MONDAY

$$\int_0^\infty \frac{\sin \alpha a \cos \alpha x}{\alpha} \, d\alpha = \frac{\pi}{2} \quad [y(x) \le a]$$

$$= 0 \quad [y(x) > a]$$

Now evaluate